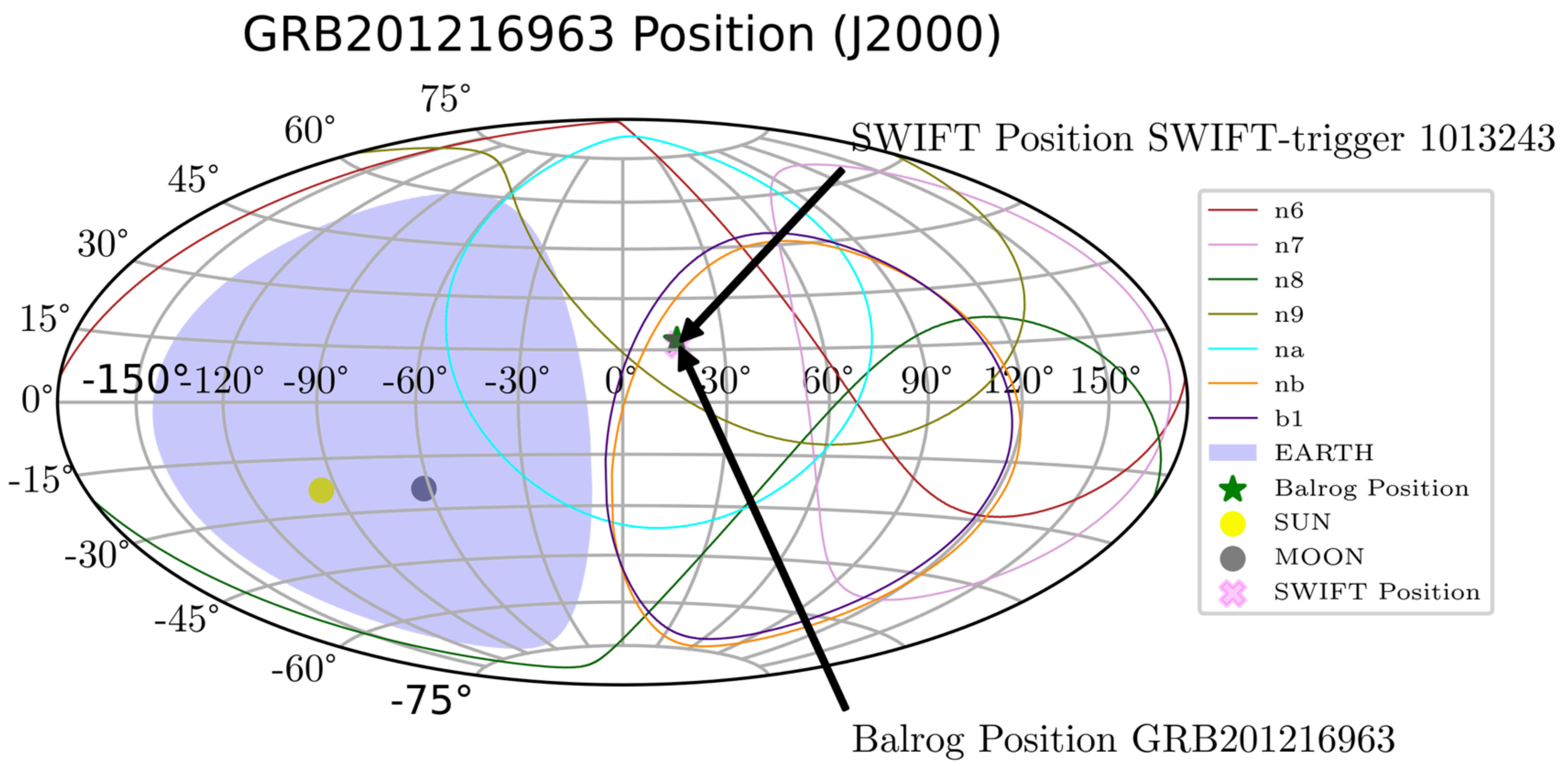
Statistical approach to gamma-ray burst localization

Triangulation via non-stationary time-series models

J. Michael Burgess

Max-Planck-Institut für
extraterrestrische Physik



gbm_drm_gen

rball

GitHub

Context

The Third Interplanetary Network

The figure on the left is an HST observation of GRB970228; on the right, the logo of the 3rd Interplanetary Network.

GRB and Magnetar Bibliography

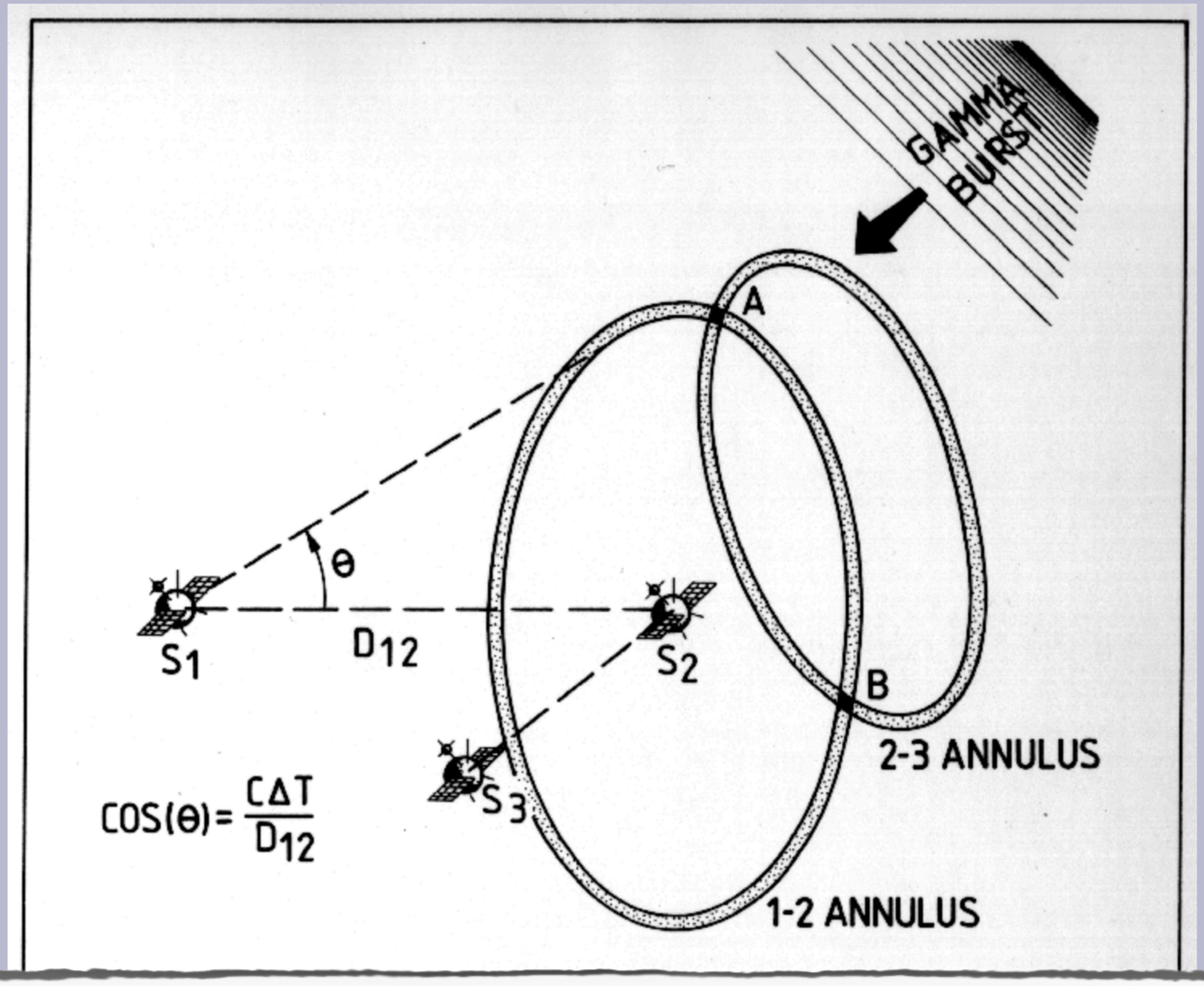
IPN Data

The Master Burst List

Triangulation Maps

Acknowledging and Referencing the Website

The third interplanetary network (IPN3) is a group of spacecraft equipped with gamma-ray burst detectors. By timing the arrival of a burst at several spacecraft, its precise location can be found. The farther apart the detectors, the more precise the location. The principle is illustrated in the figure below. Each pair of spacecraft, like S1 and S2, gives an annulus of possible arrival directions whose center is defined by the vector joining the two spacecraft, and whose radius theta depends on the difference in the arrival times divided by the distance between the two spacecraft.

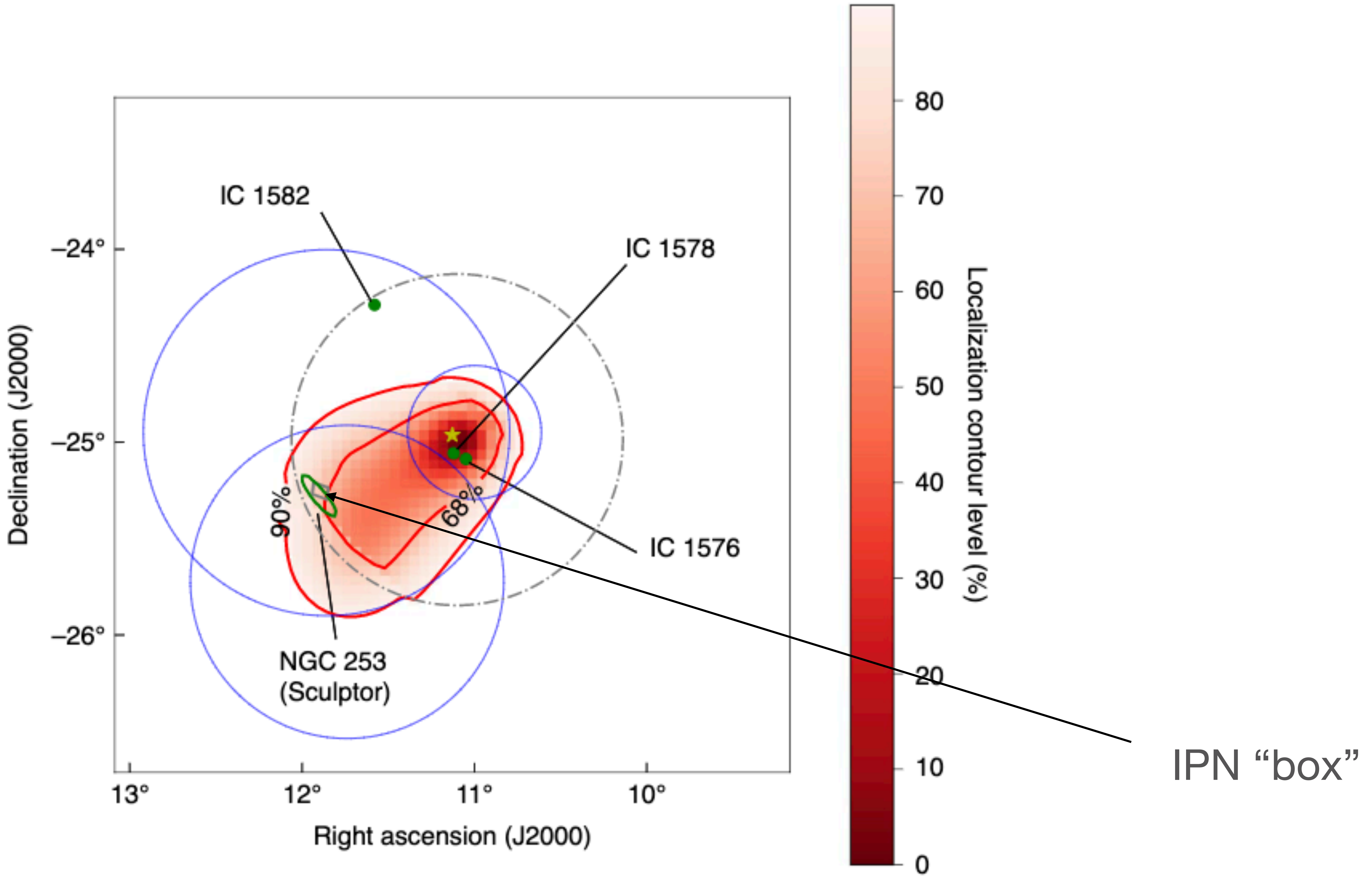


Context

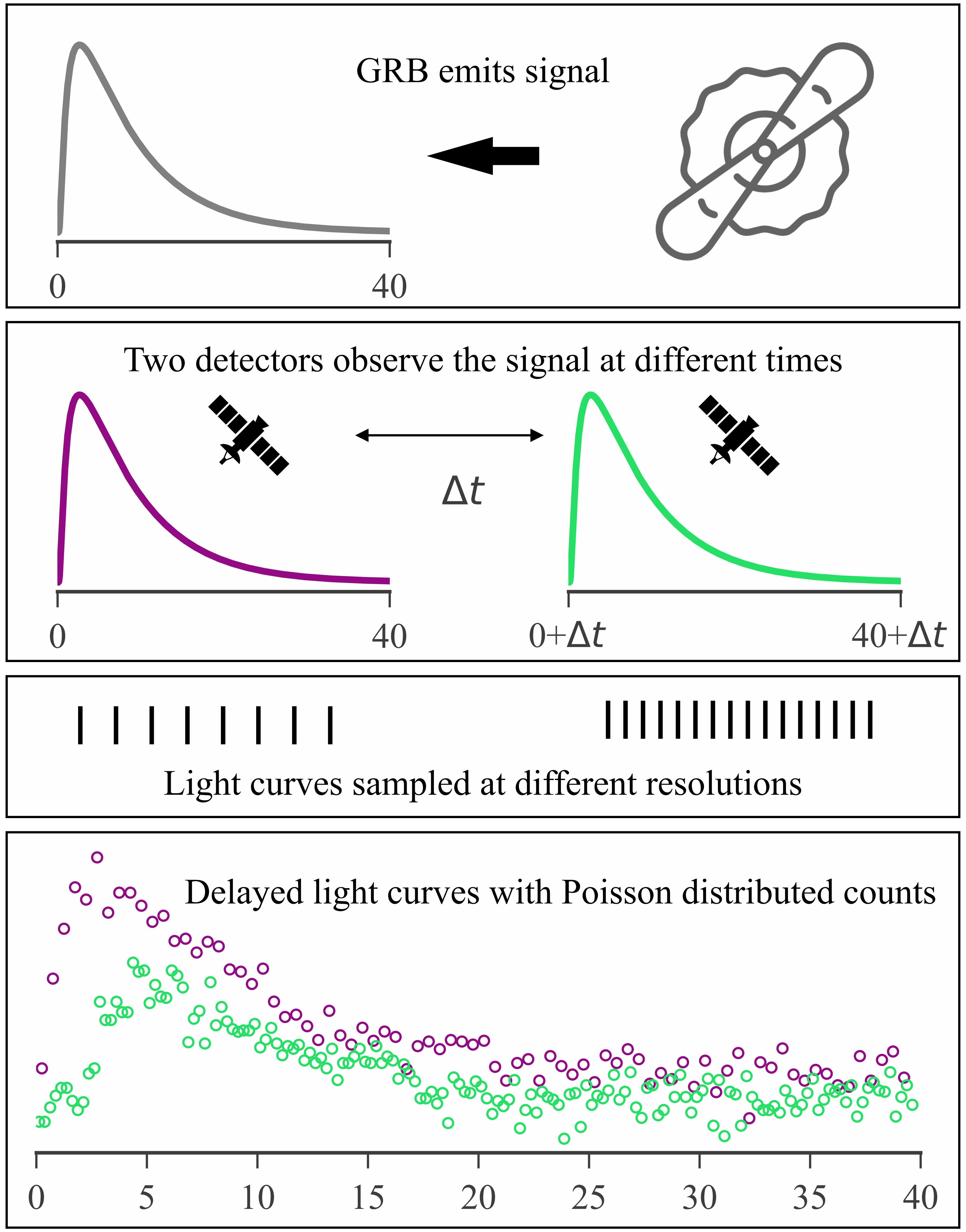
nature astronomy
LETTERS
https://doi.org/10.1038/s41550-020-01287-8

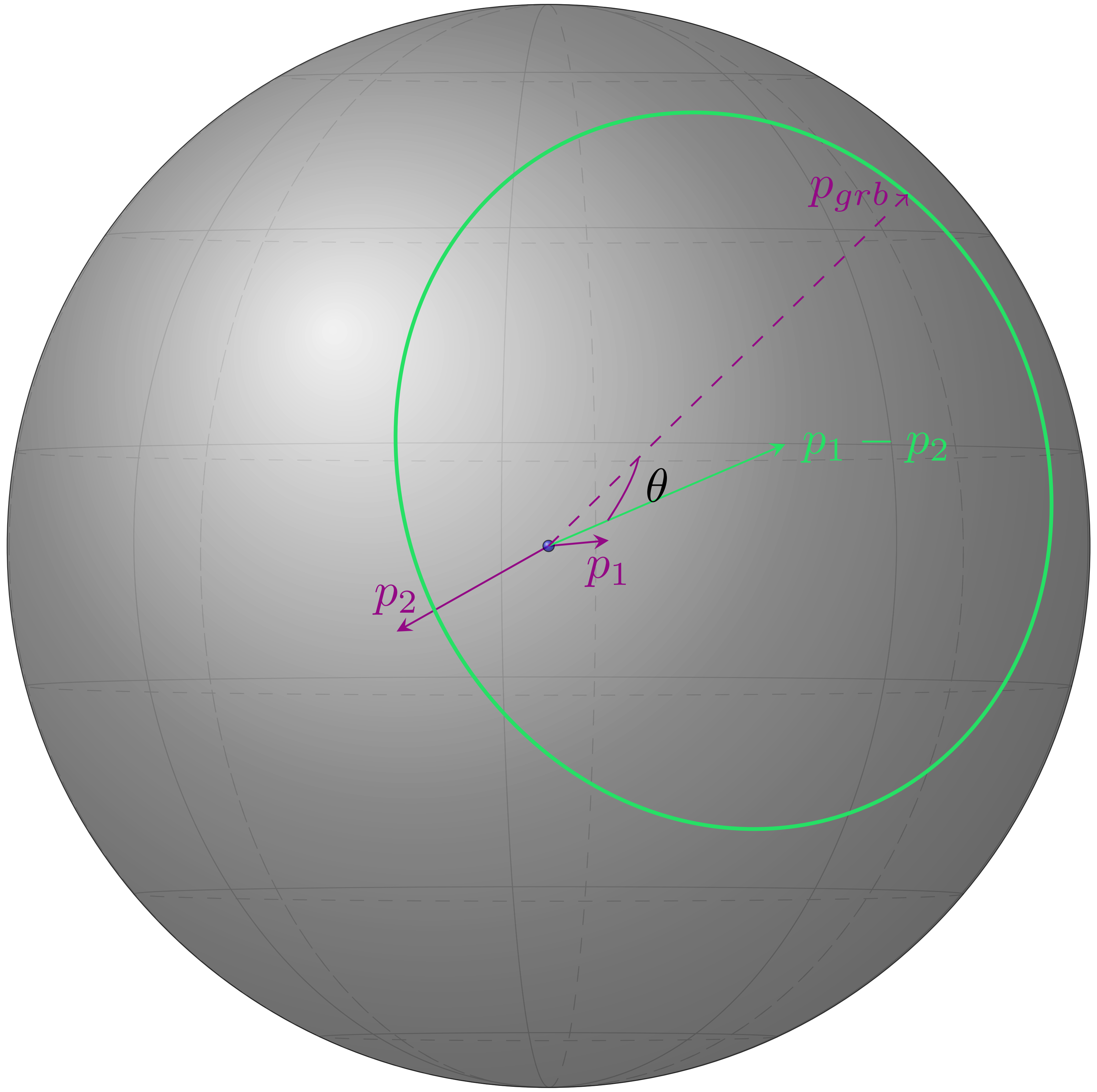
High-energy emission from a magnetar
in the Sculptor galaxy

The Fermi-LAT Collaboration



Context



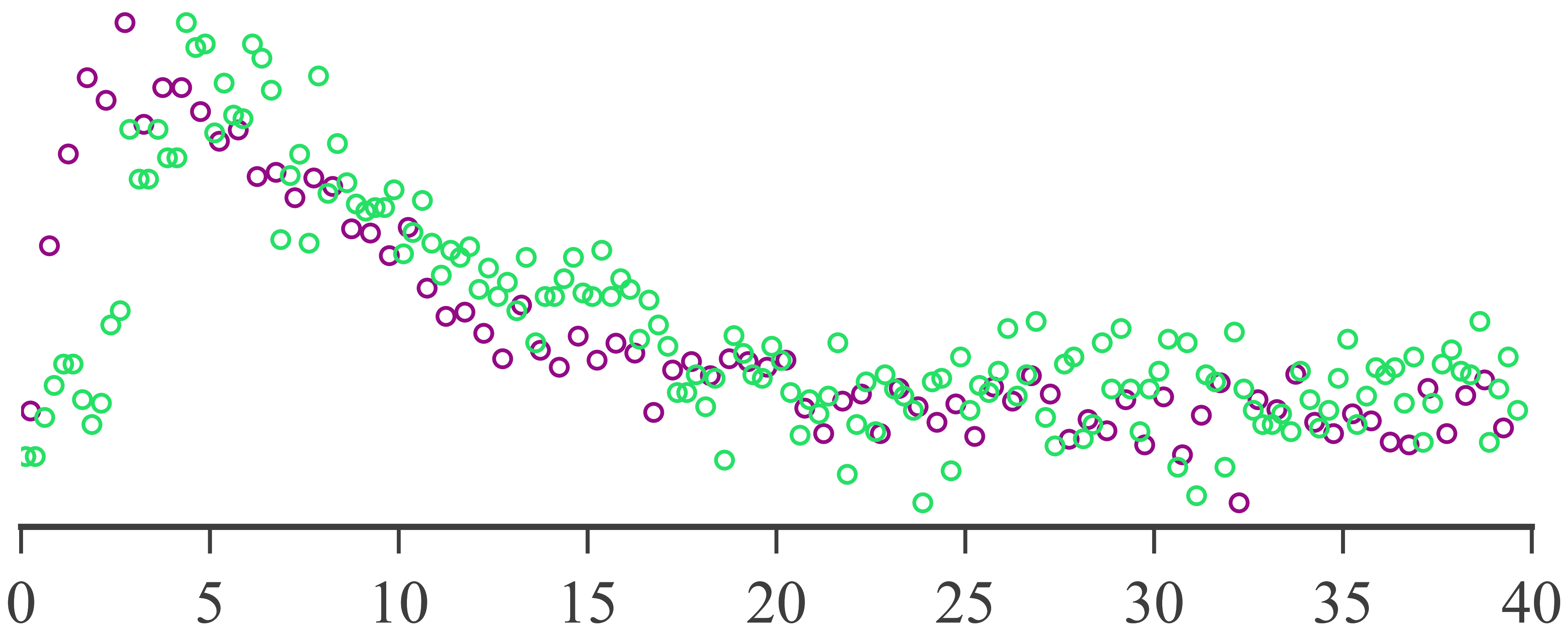


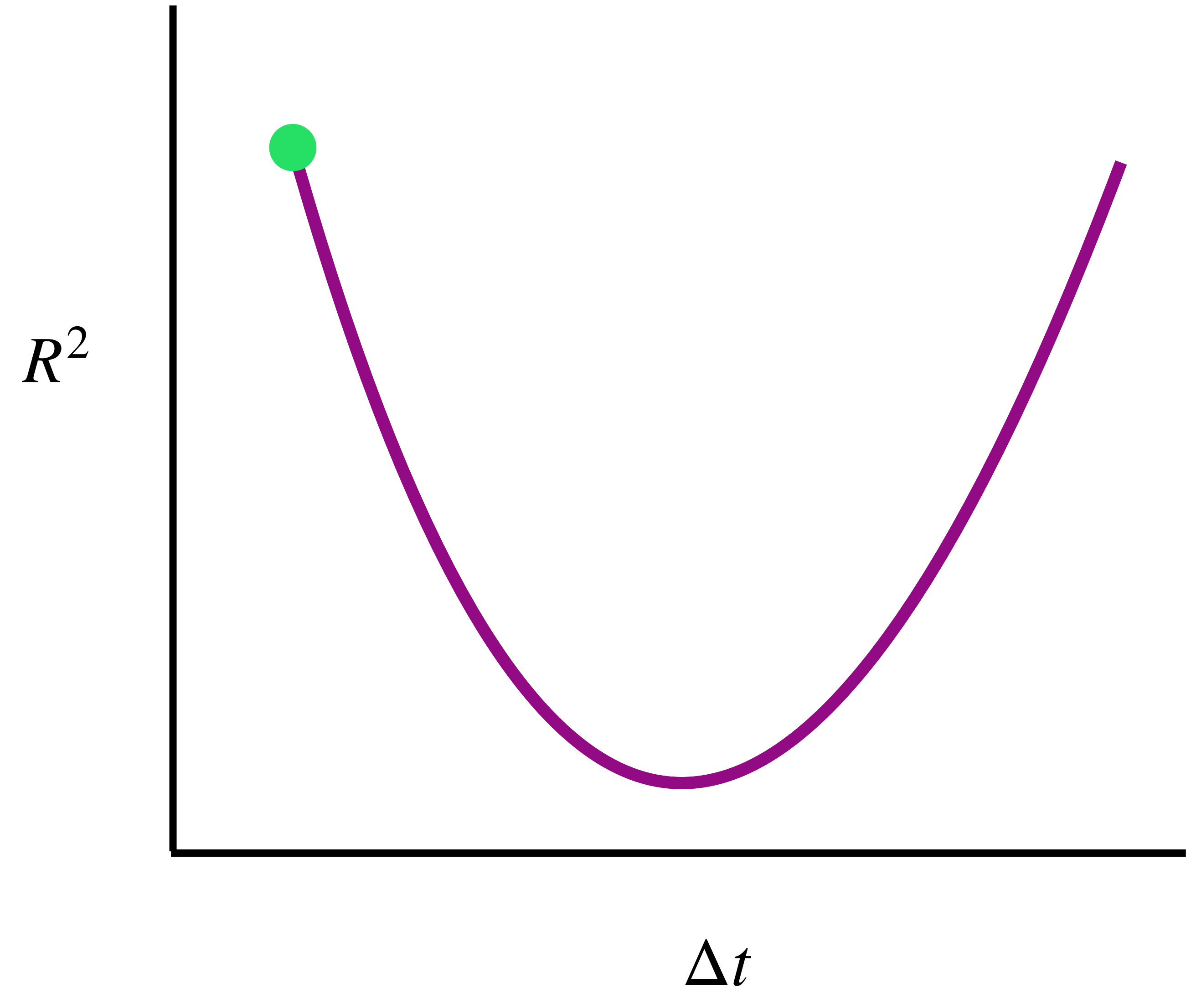
$$\Delta t\left(p_{\mathrm{grb}}, p_1, p_2\right) = \frac{p_{\mathrm{grb}} \cdot \left(p_1 - p_2\right)}{c}$$

$$\theta = \cos^{-1}\left(\frac{c\Delta t}{\| p_1 - p_2 \|}\right)$$

Concept

$$R^2(\Delta t \equiv j\delta) = \sum_{i=i_{\text{start}}}^{i=i_{\text{start}}+N} \frac{\left(\tilde{c}_{2,i} - s\tilde{c}_{1,i+j}\right)^2}{\left(\sigma^2_{2,i} + s^2\sigma^2_{1,i+j}\right)}$$





Cross-correlation

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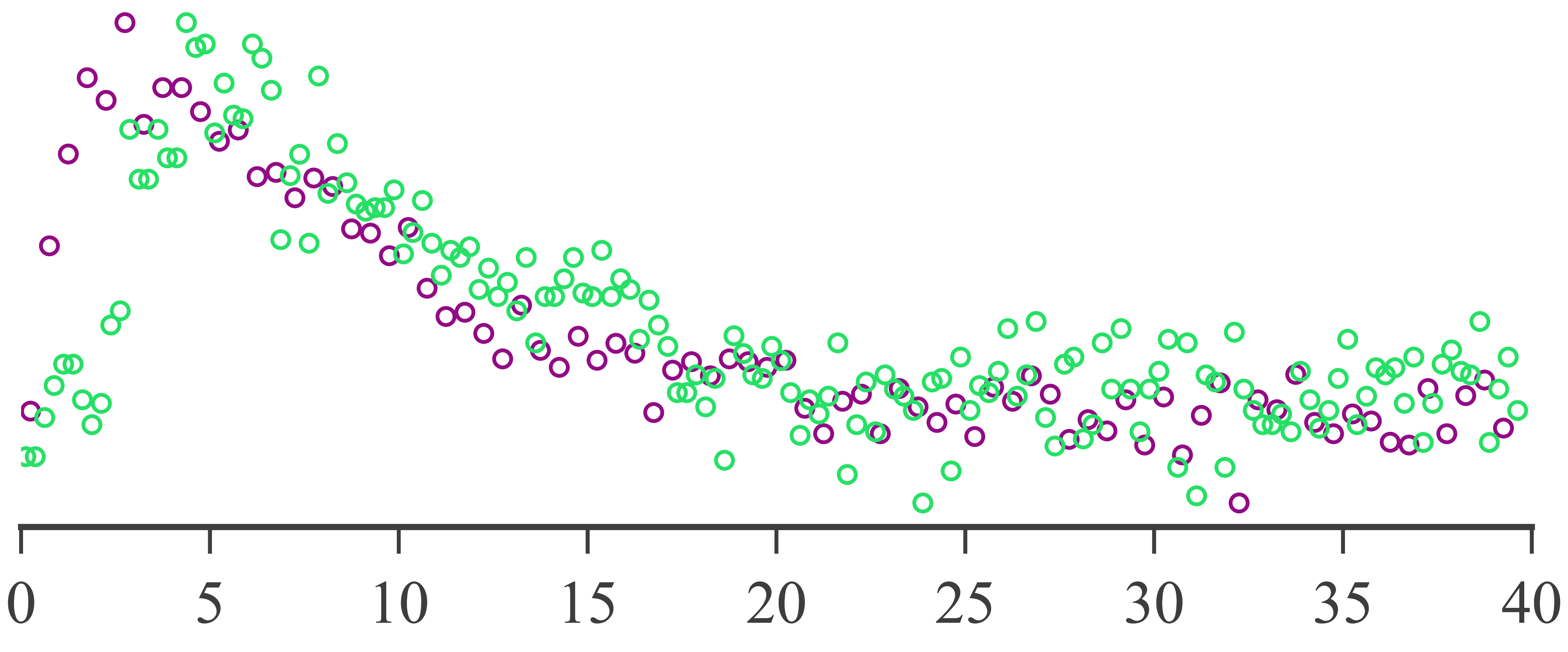

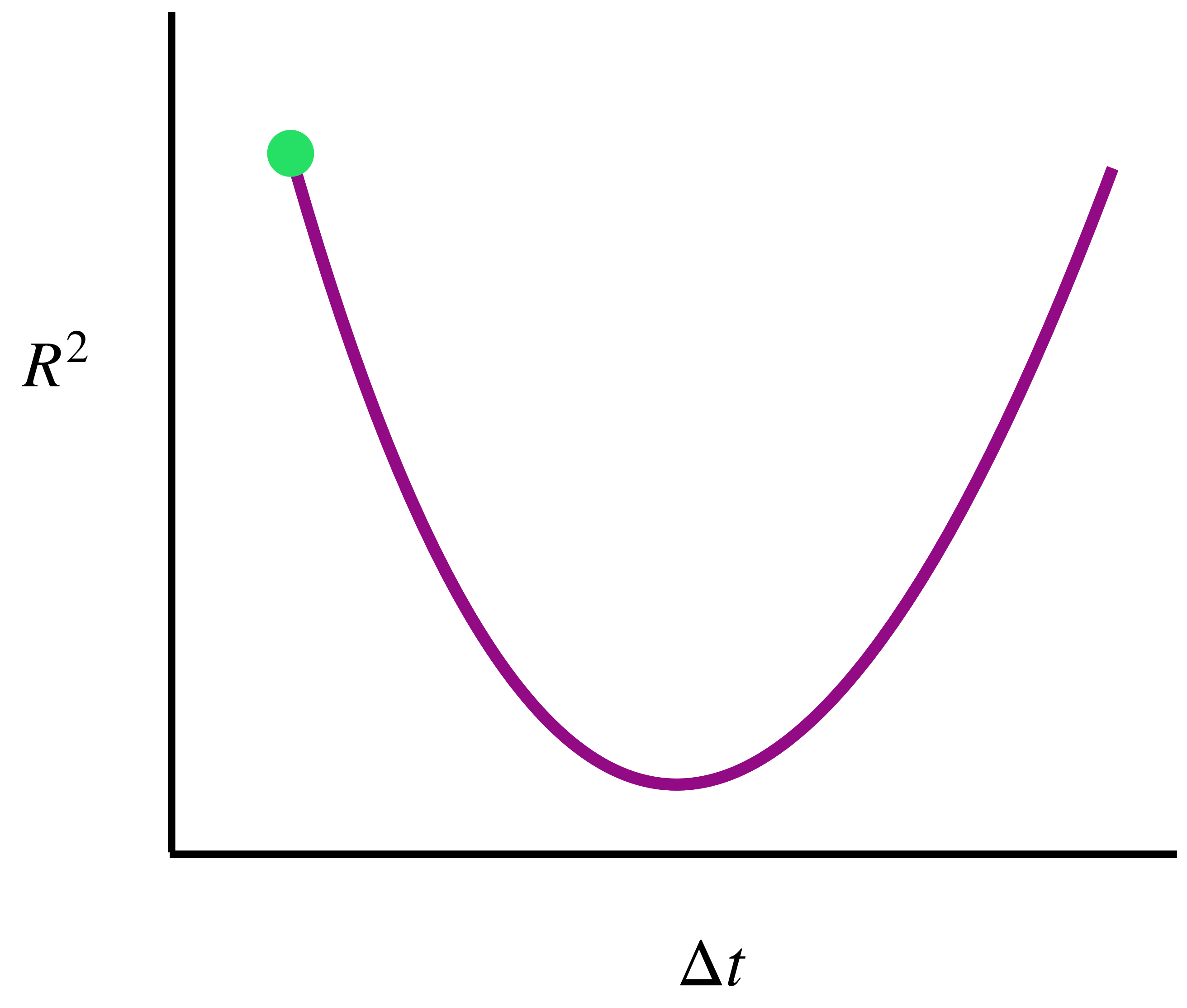

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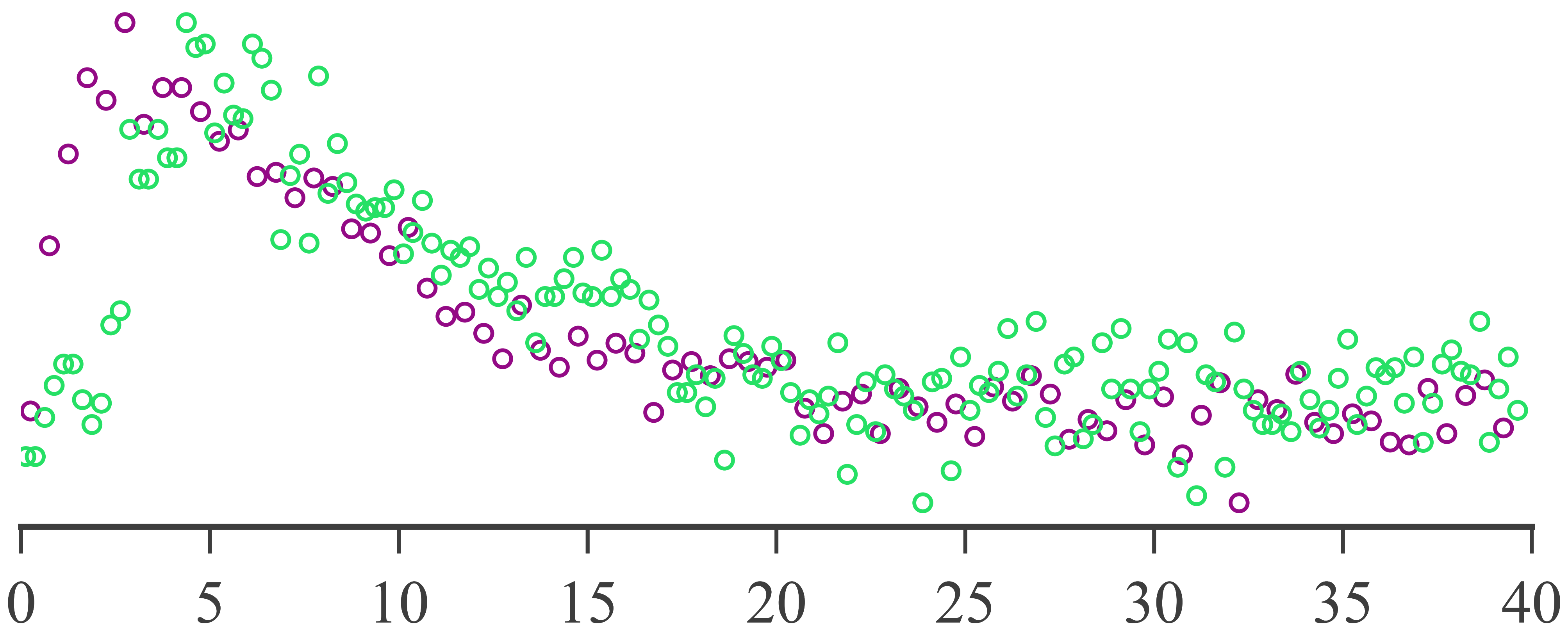

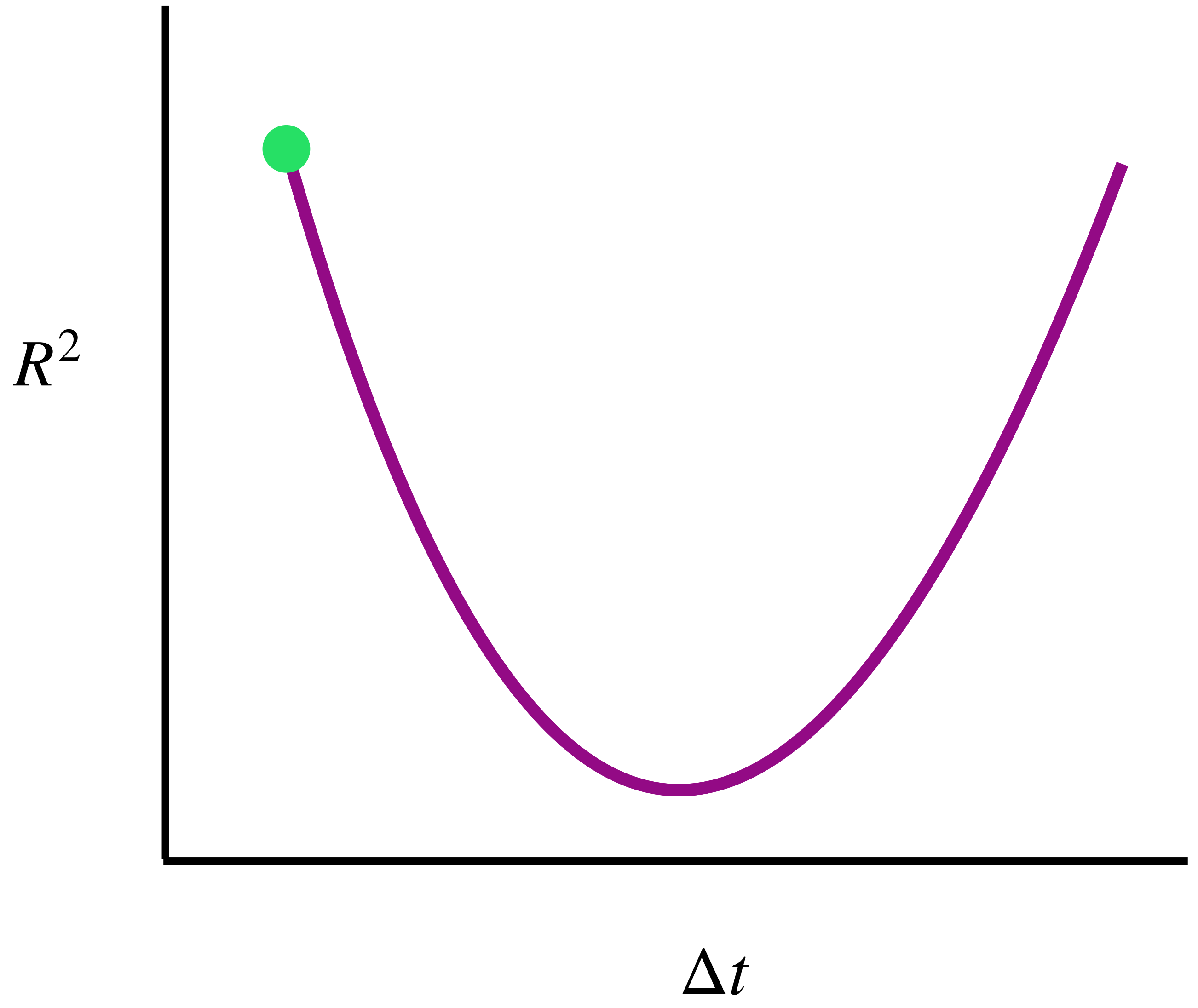

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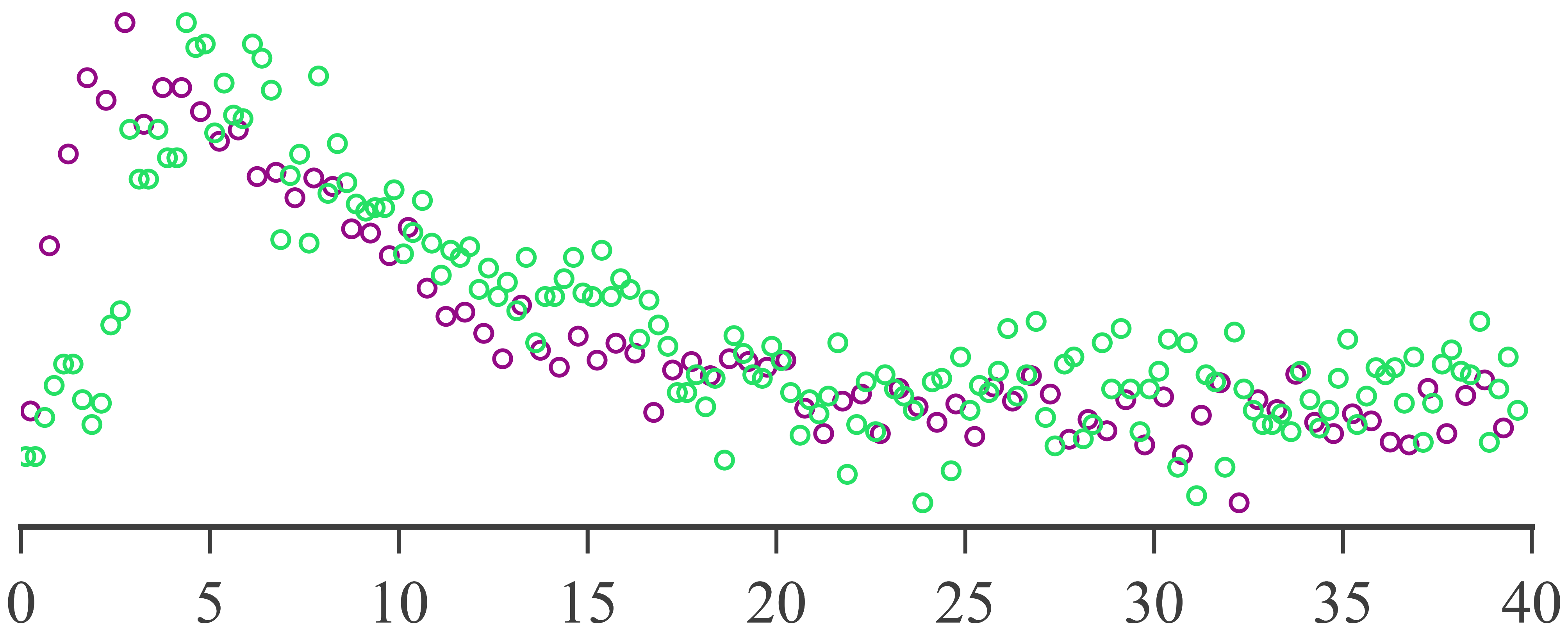

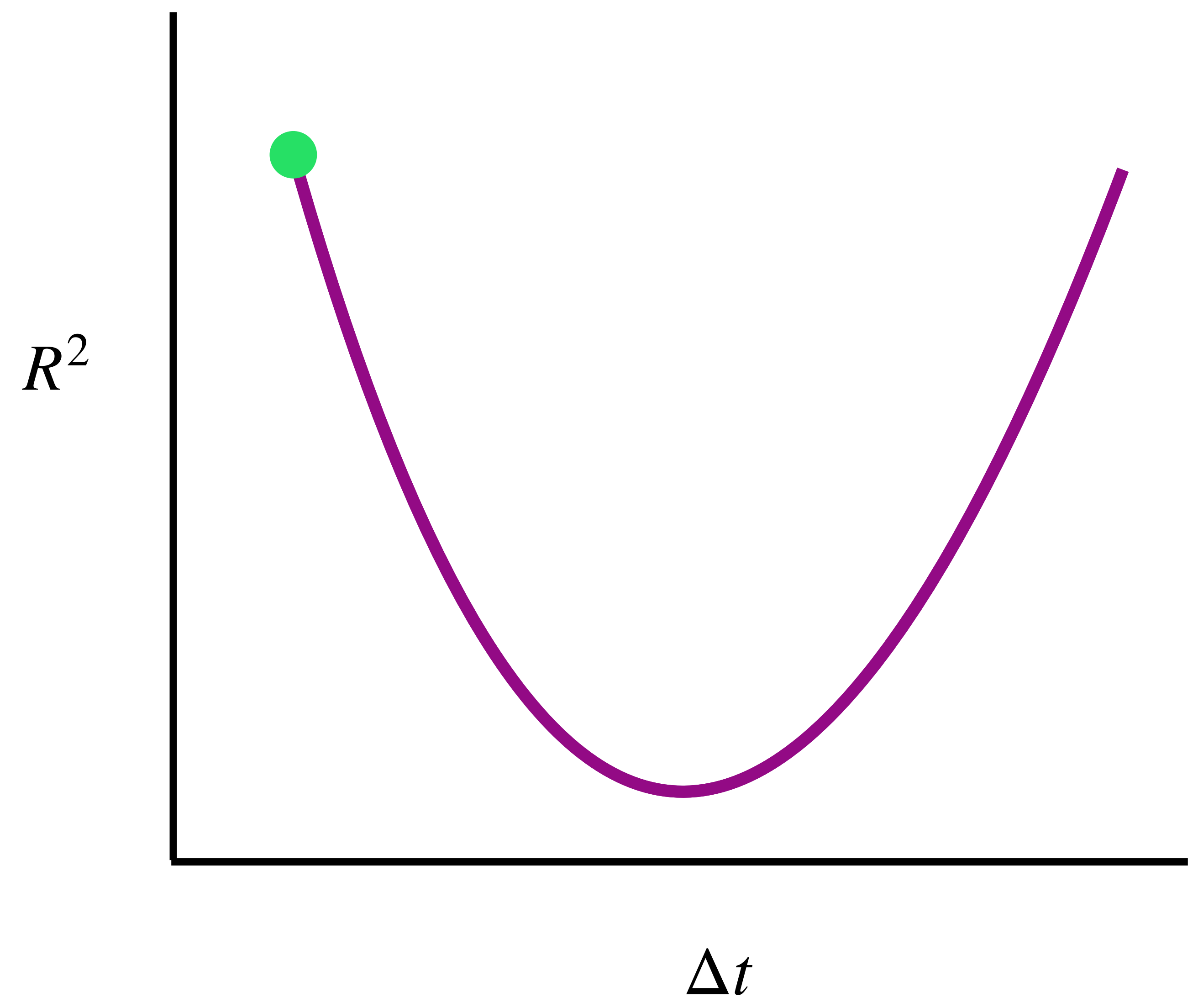

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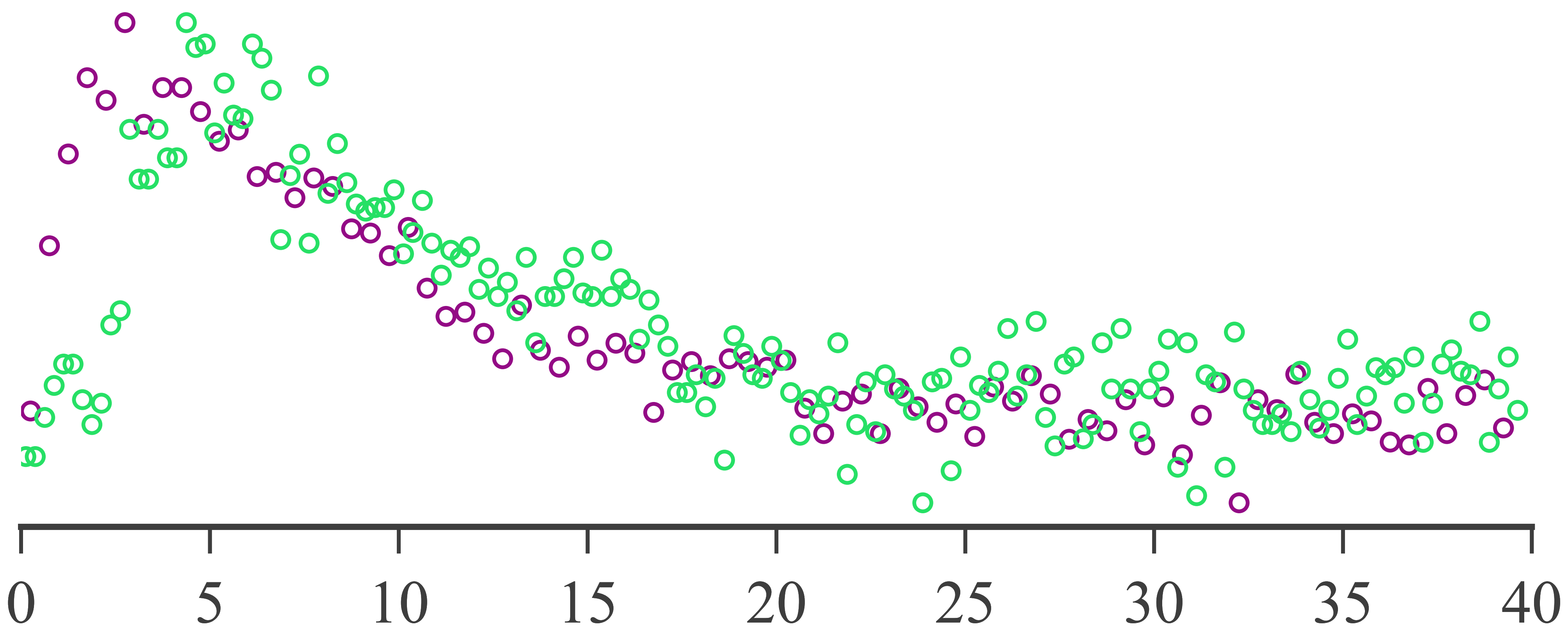

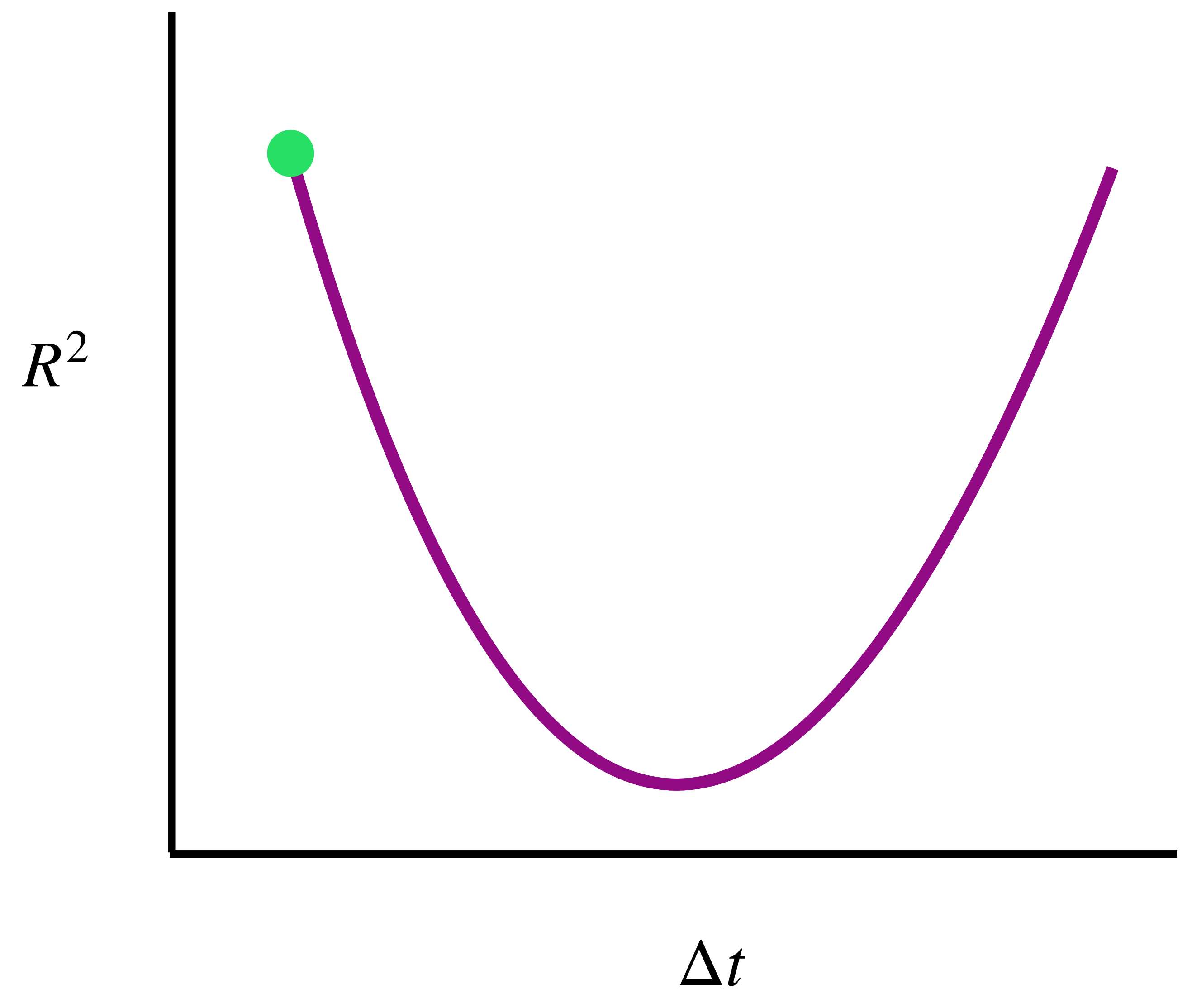

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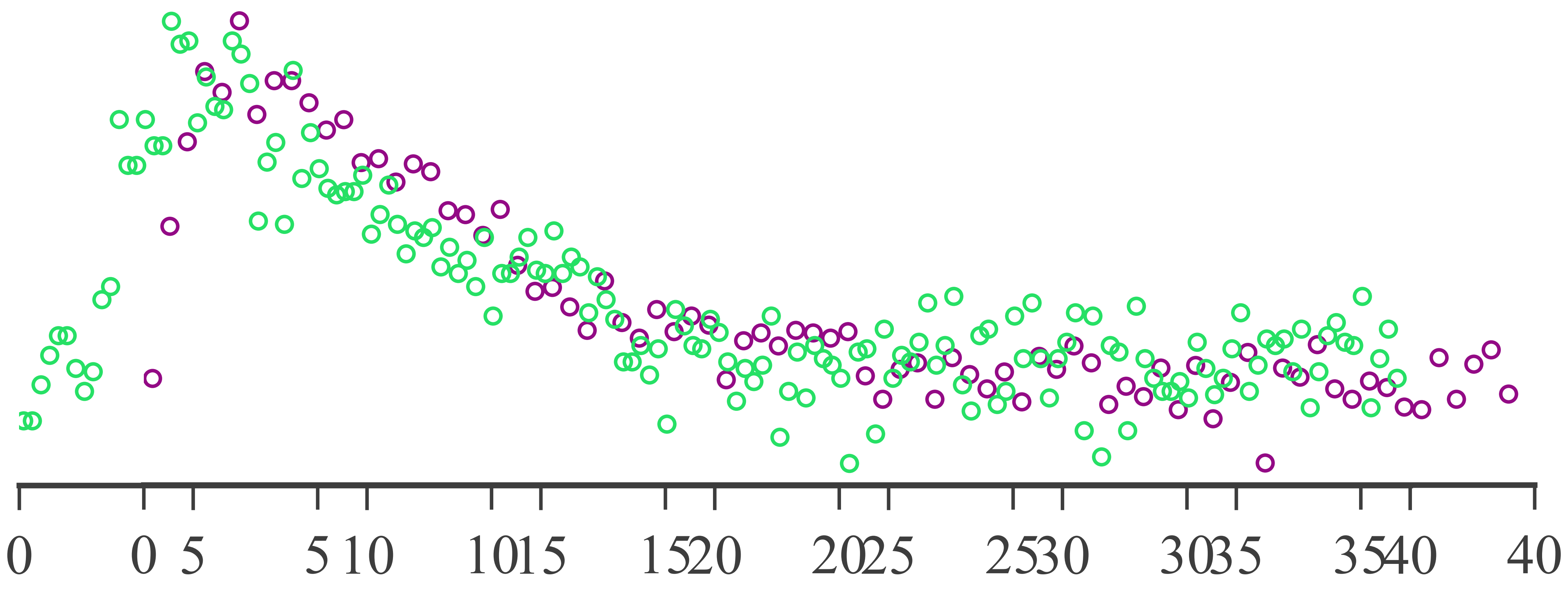
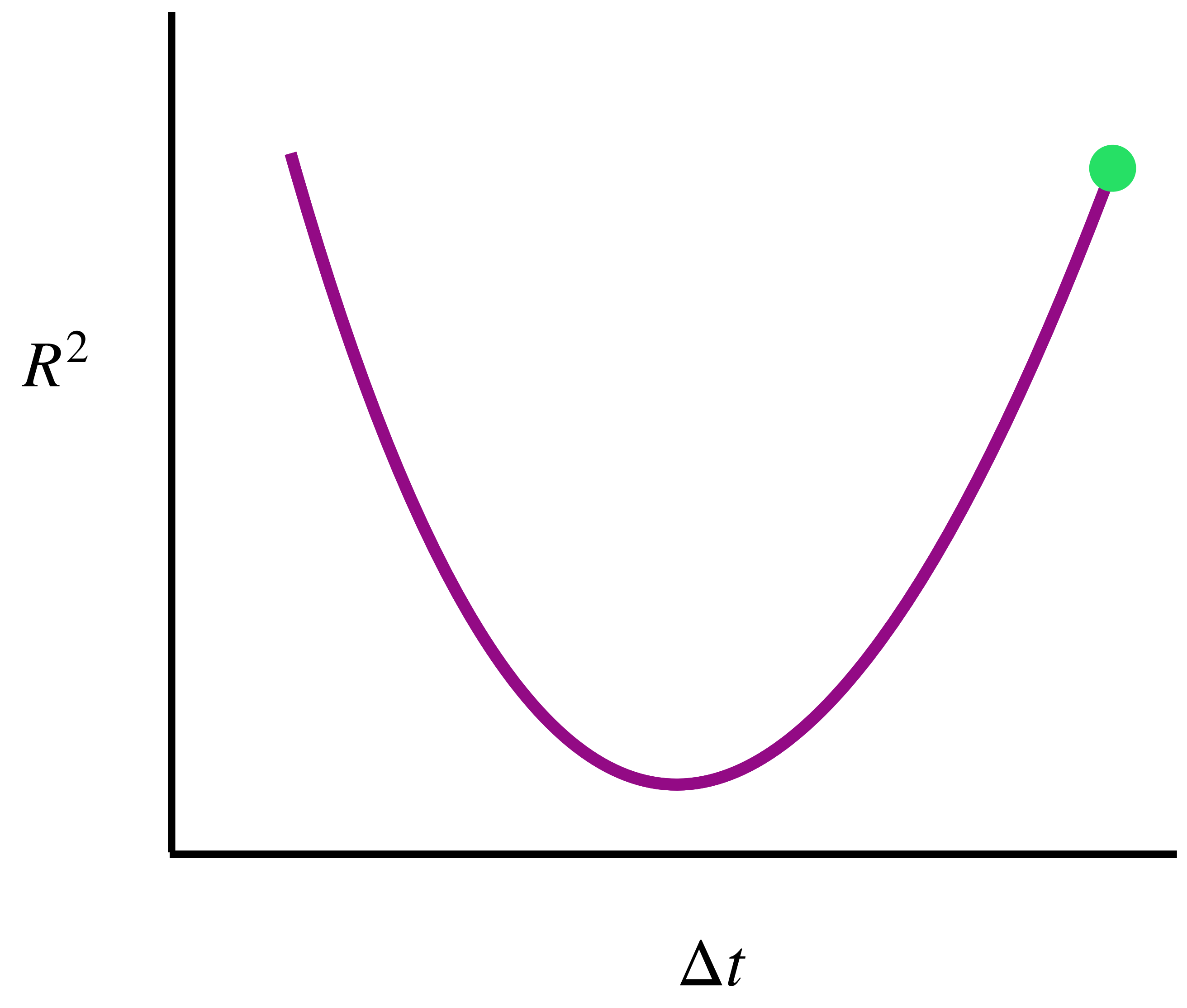
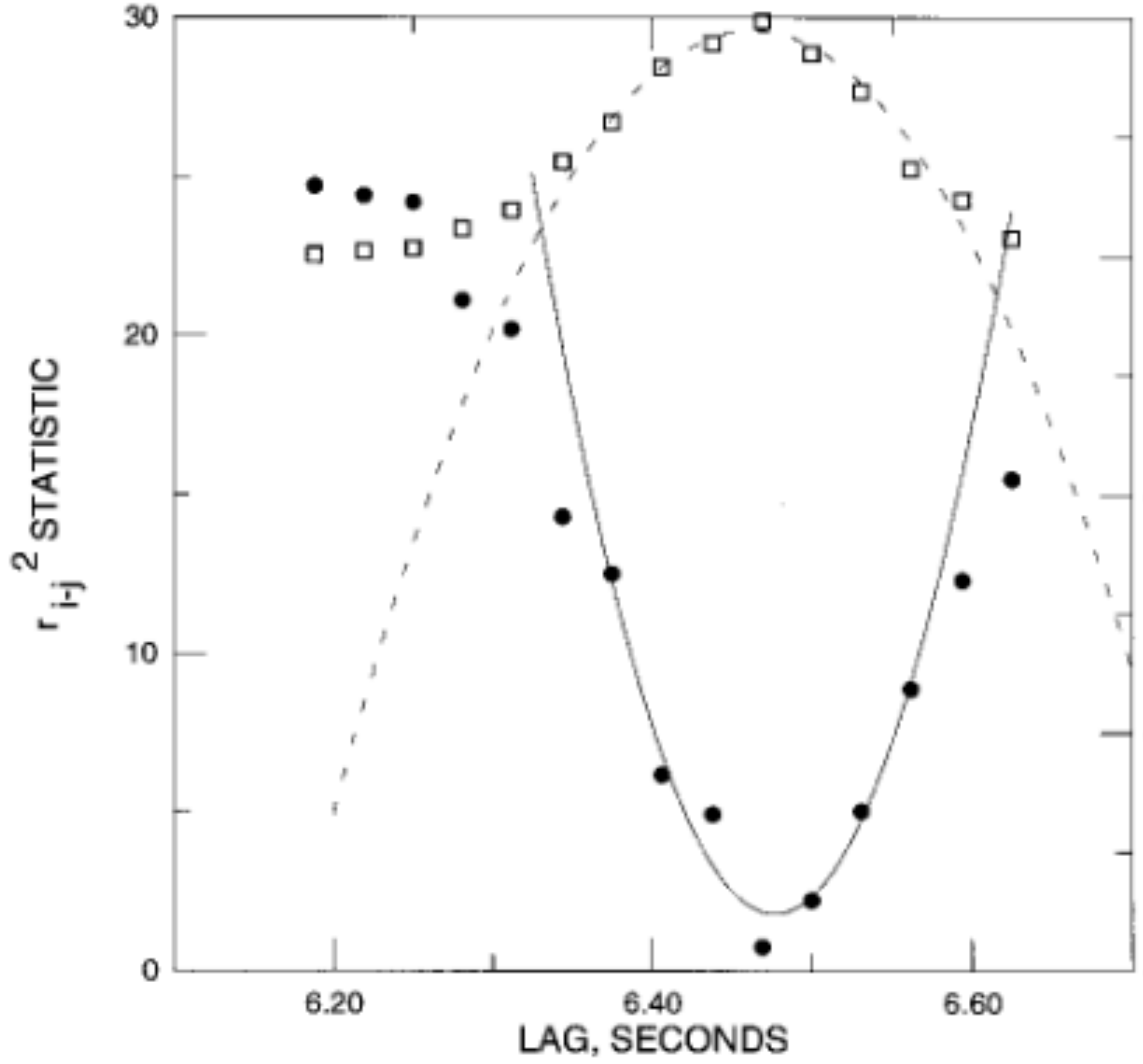
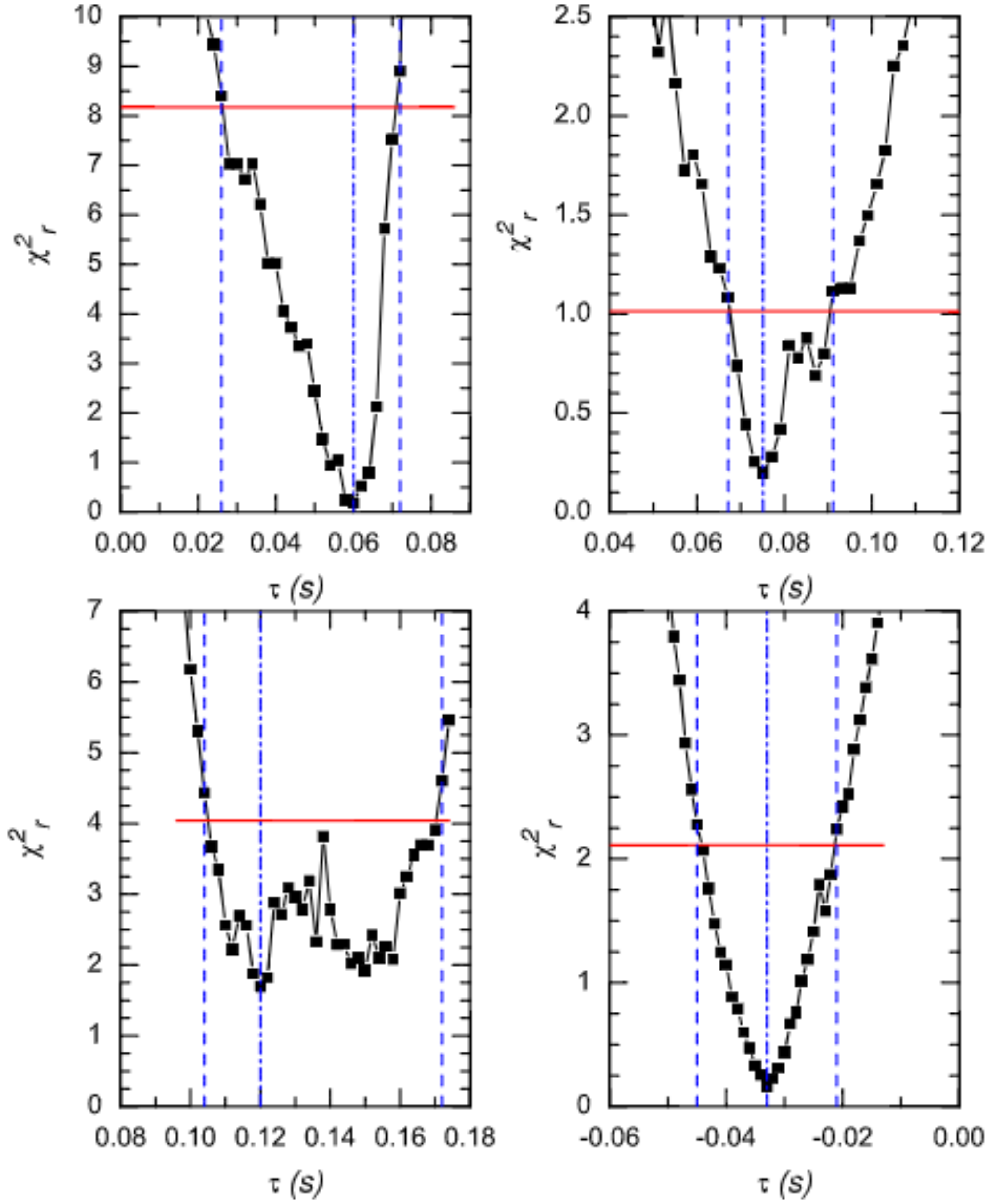




Cross-correlation



Hurley et al. 1992-2013



Cross-correlation

THE ASTROPHYSICAL JOURNAL, 333:646–659, 1988 October 15

THE DISCRETE CORRELATION FUNCTION: A NEW METHOD FOR ANALYZING UNEVENLY SAMPLED VARIABILITY DATA

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AND

J. H. KROLIK
Department of Physics and Astronomy, Johns Hopkins University

Received 1987 December 29; accepted 1988 April 12

ABSTRACT

A method of measuring correlation functions without interpolating in the temporal domain, "the discrete correlation function," is introduced. It provides an assumption-free representation of the correlation measured in the data and allows meaningful error estimates. This method avoids the problem of spurious correlations at zero lag due to correlated errors. It is shown that physical interpretation of the cross-correlation function of two series believed to be related by a convolution requires knowledge of the input function's fluctuation power spectrum. In the case of AGN line-continuum cross-correlation functions, the interpretation also involves model dependence in the form of symmetry assumptions and must take into account intrinsic scale bias. Application to published data for Akn 120 and NGC 4151 illustrates this method's capabilities. No correlation was found for the optical data for Akn 120, but the ultraviolet NGC 4151 data show a strong correlation, indicating that the broad C IV feature emanates from a region whose size is greater than 1.2 and less than 20 light-days. These bounds on the size of the line-emitting region in NGC 4151 are in good agreement with the predictions of photoionization models.

Subject headings: galaxies: individual (NGC 4151, Akn 120) — galaxies: Seyfert — numerical methods — quasars — radio sources: variable

Cross-correlation

Beware of statistical “methods” developed by astronomers!

(This includes the 2DKS test)

Cross-correlation

Signal model

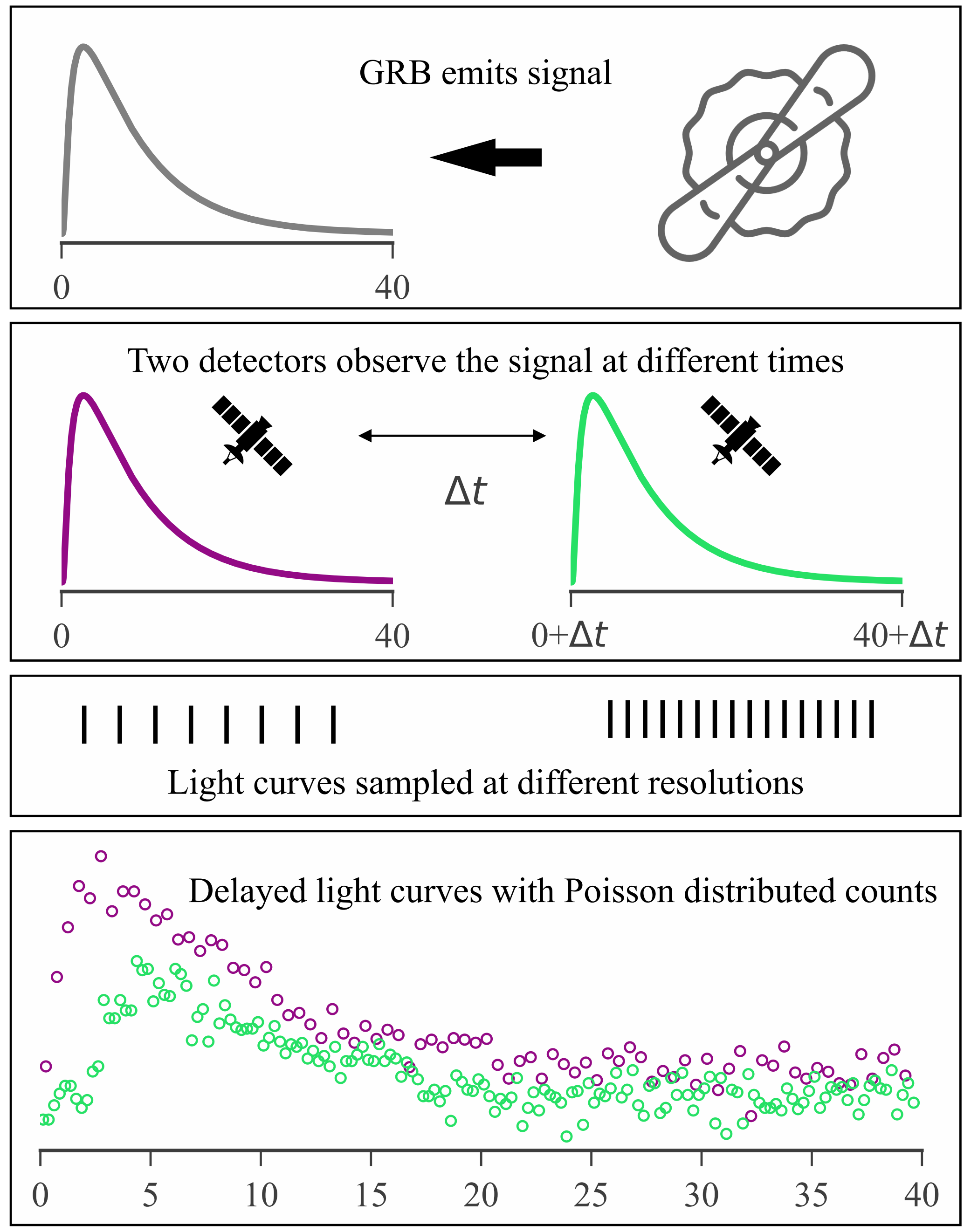
$$C_i(t) = A_i S_i(t) + B_i(t)$$

Likelihood per detector

$$\mathscr{L}\left(c_i(t) \mid C_i(t)\right) = \prod_{j=1,\ldots,N_{\text{bins}}} \mathscr{P}\left(\int_{t_0+(j-1)\delta t}^{t_0+j\delta t} dt' C_i(t')\right)[c_i(t)]$$

Total likelihood

$$\mathscr{L}\left(\{c_i(t)\}_{i=1,\ldots,N_{\text{det}}} \mid \{C_i(t)\}_{i=1,\ldots,N_{\text{det}}}\right) = \prod_{i=1,\ldots,N_{\text{det}}} \mathscr{L}\left(c_i(t) \mid C_i(t)\right)$$



Random Fourier Features

$$\log S\left(t \mid b_1, b_2, \sigma_1, \sigma_2, \omega_1, \omega_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2\right) = \left[\sum_{i=1}^{2} \frac{\sigma_i}{\sqrt{k}} \cos\left(b_i \boldsymbol{\omega}_i \otimes t\right)\right] \cdot \boldsymbol{\beta}_1 + \left[\sum_{i=1}^{2} \frac{\sigma_i}{\sqrt{k}} \sin\left(b_i \boldsymbol{\omega}_i \otimes t\right)\right] \cdot \boldsymbol{\beta}_2$$

Generative model

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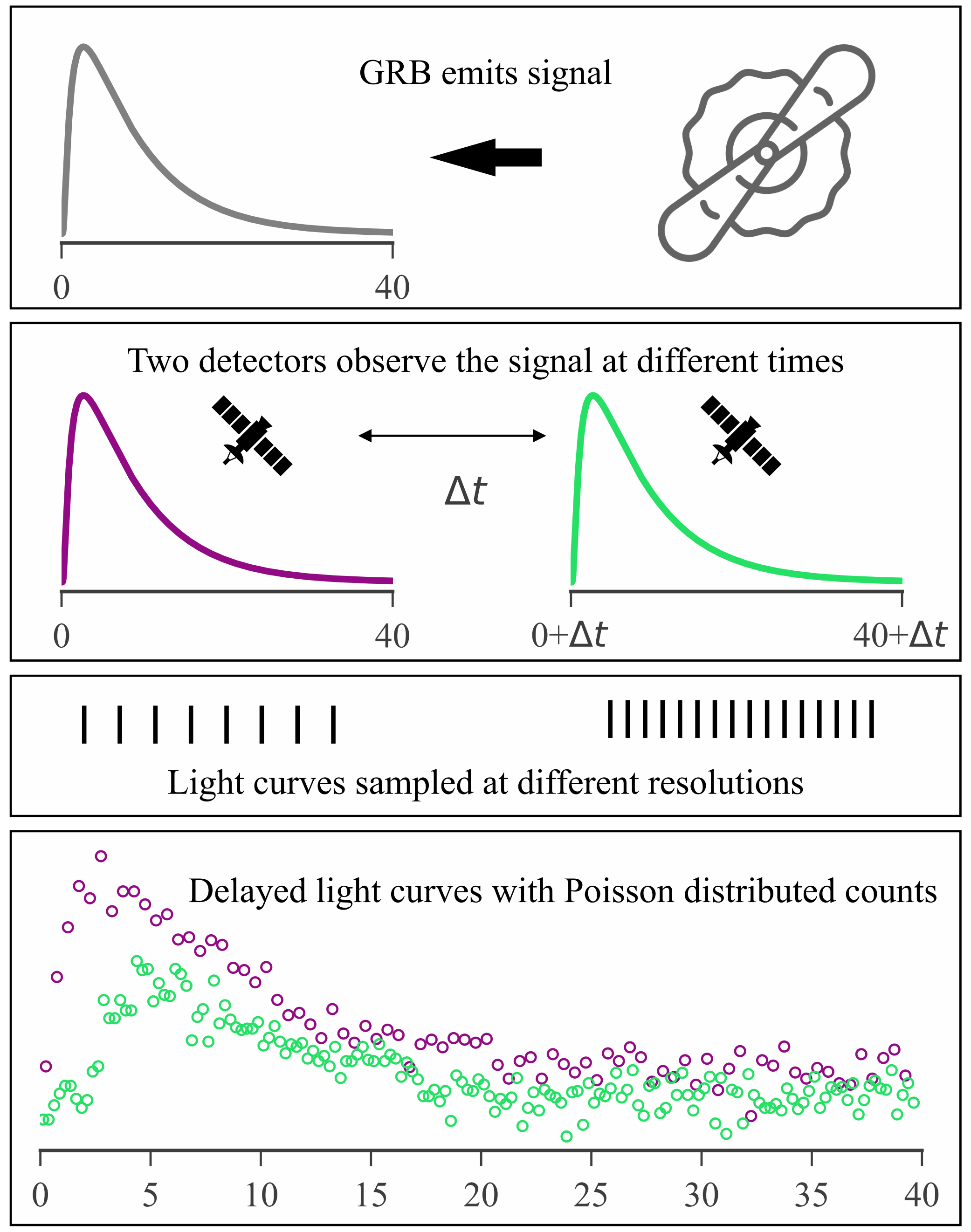

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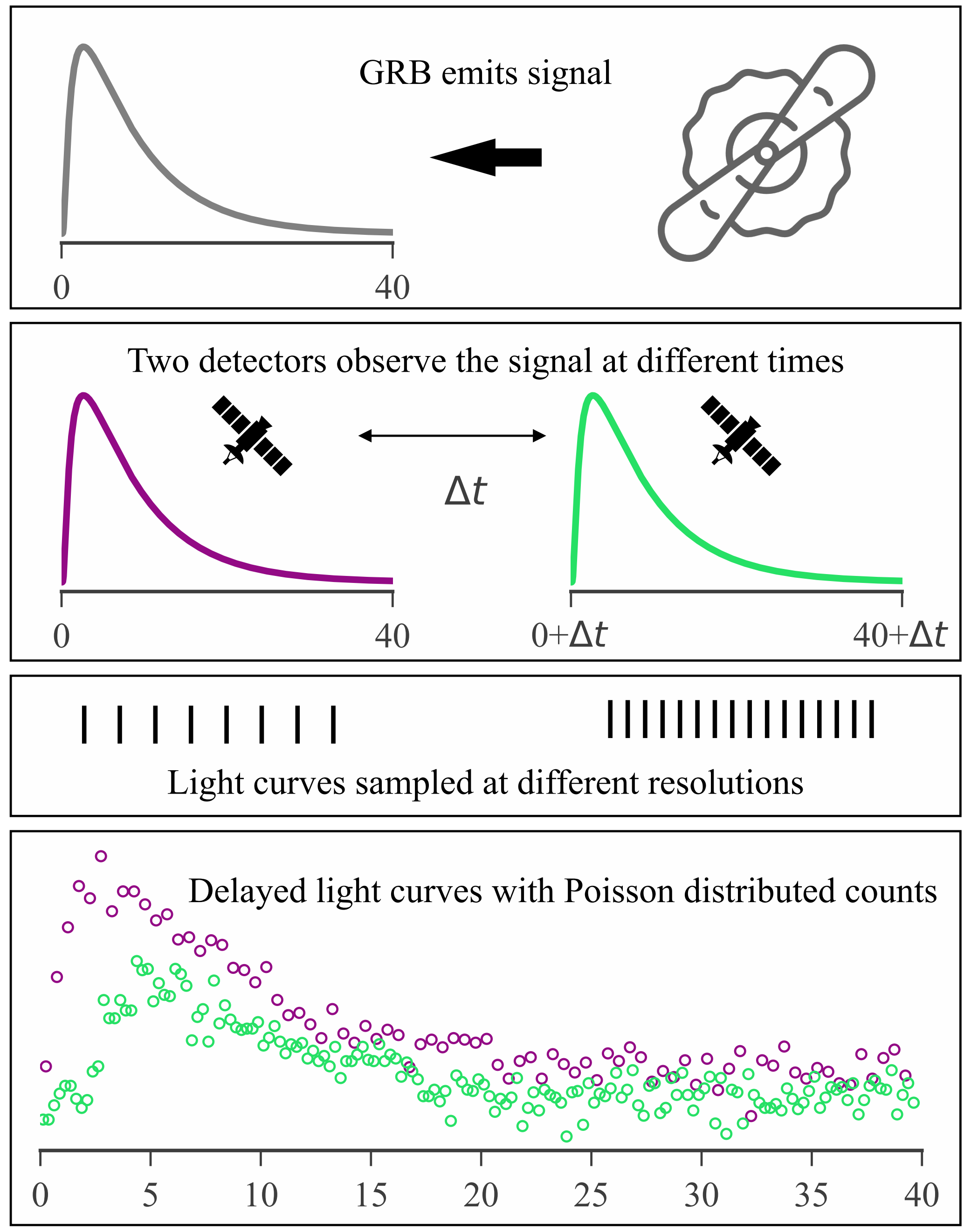

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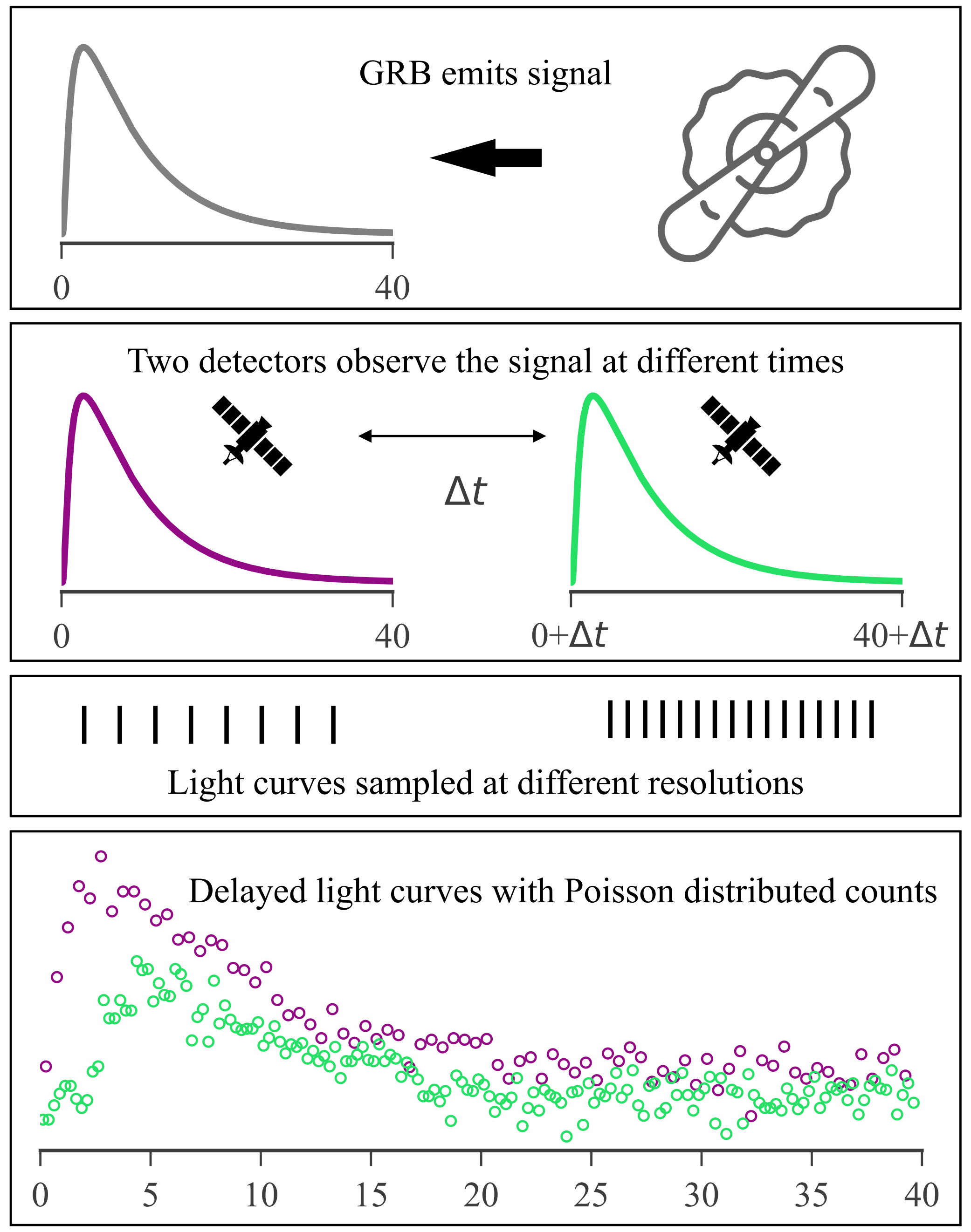
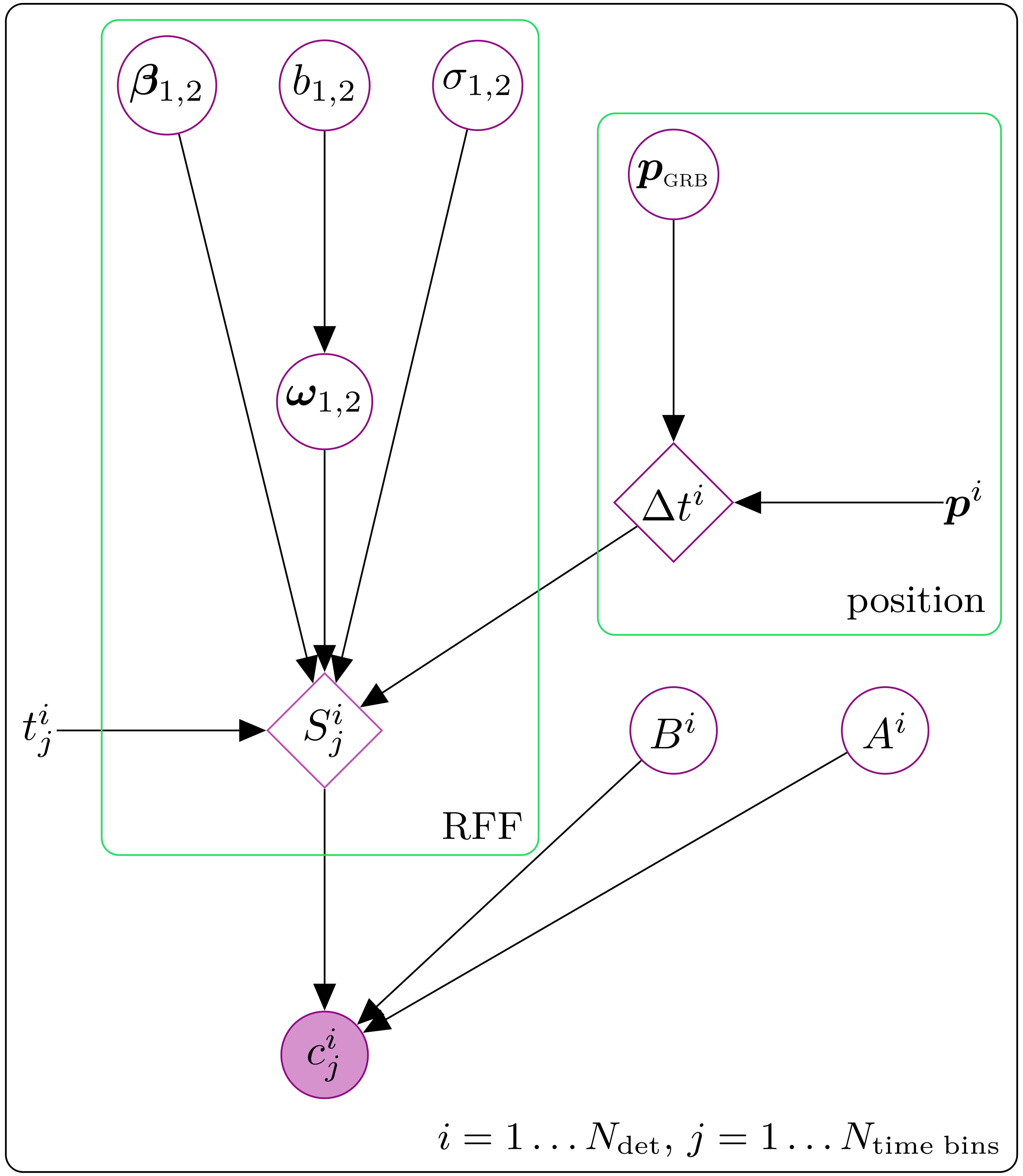
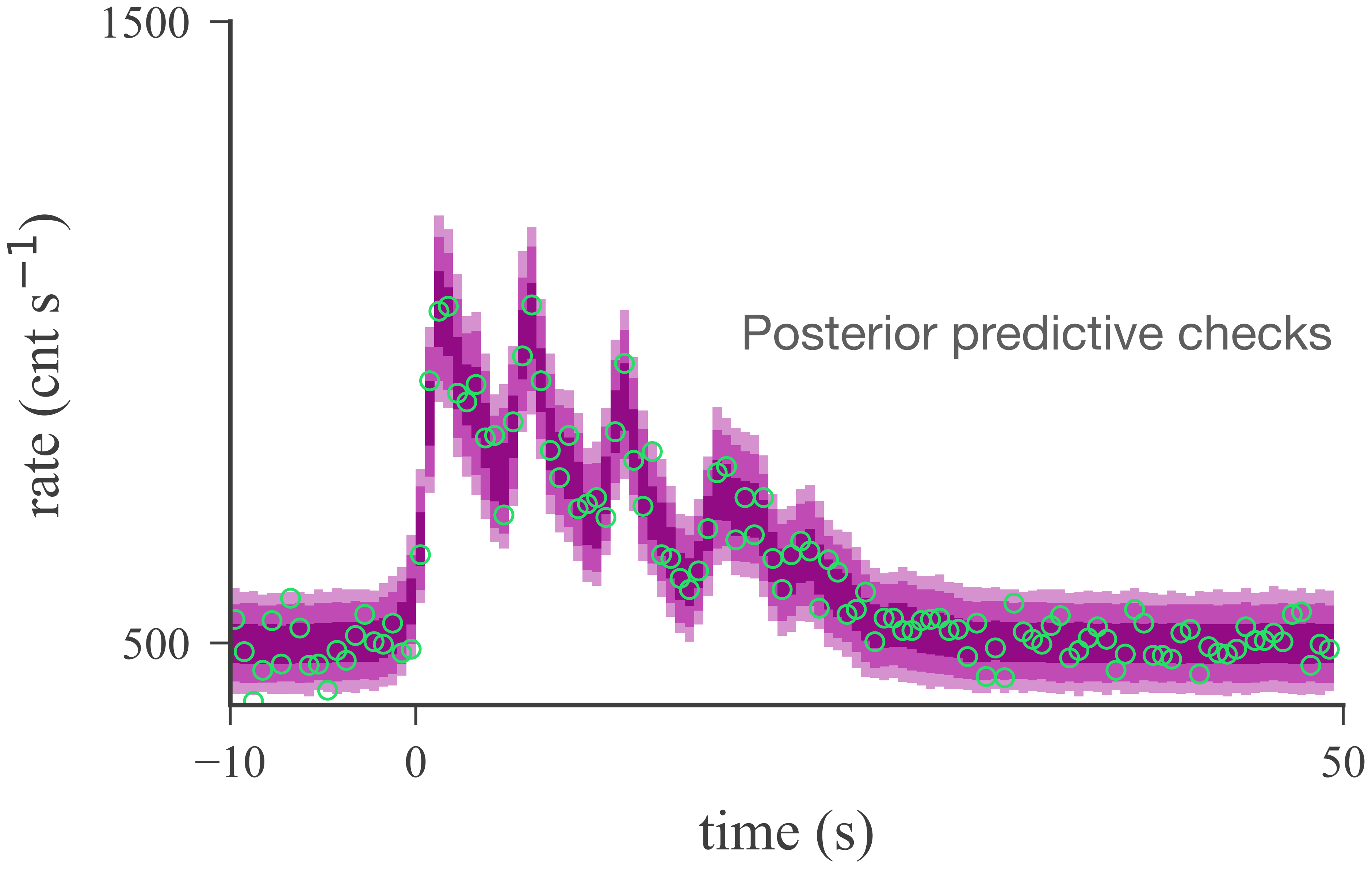
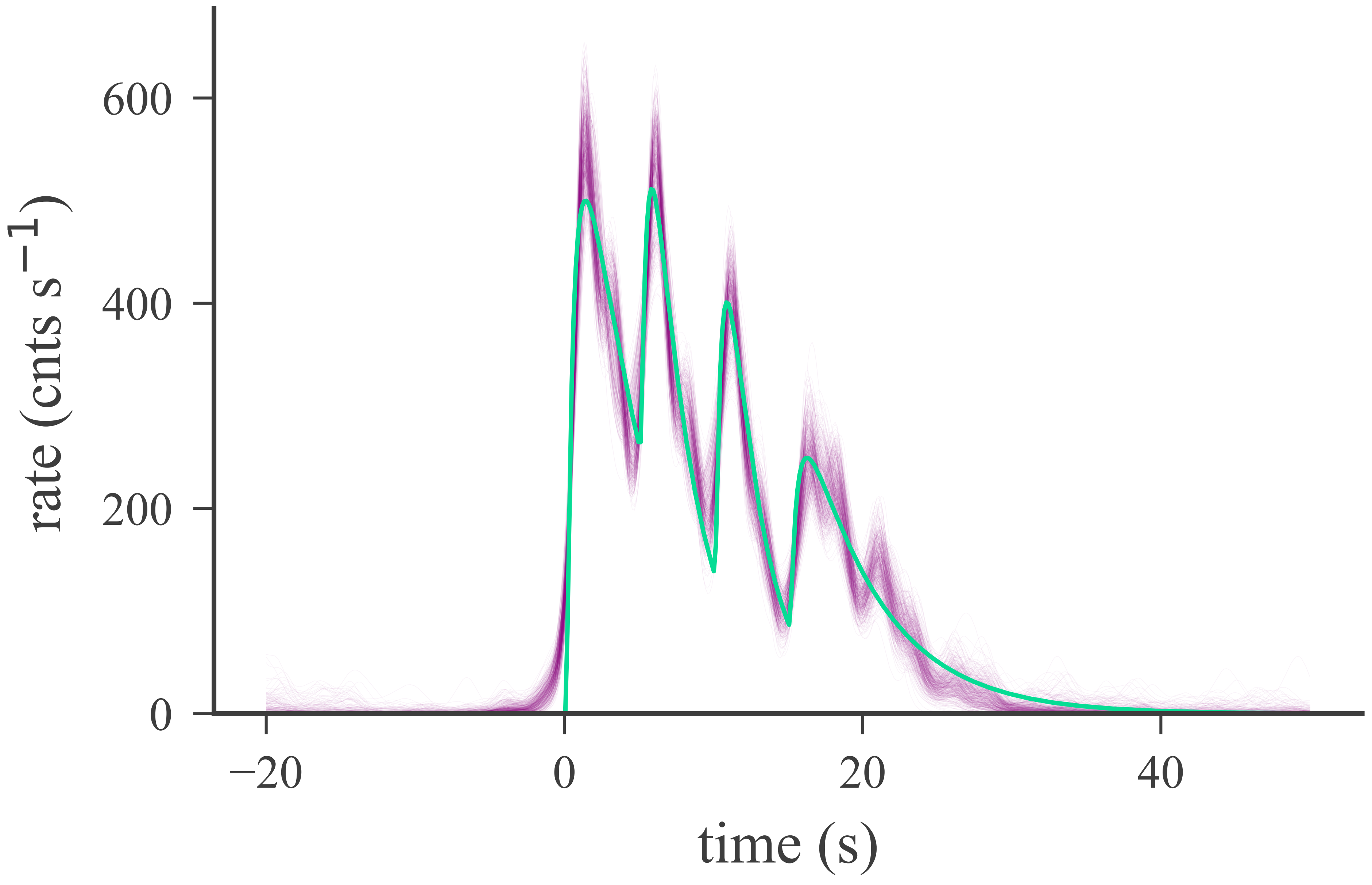


Generative model







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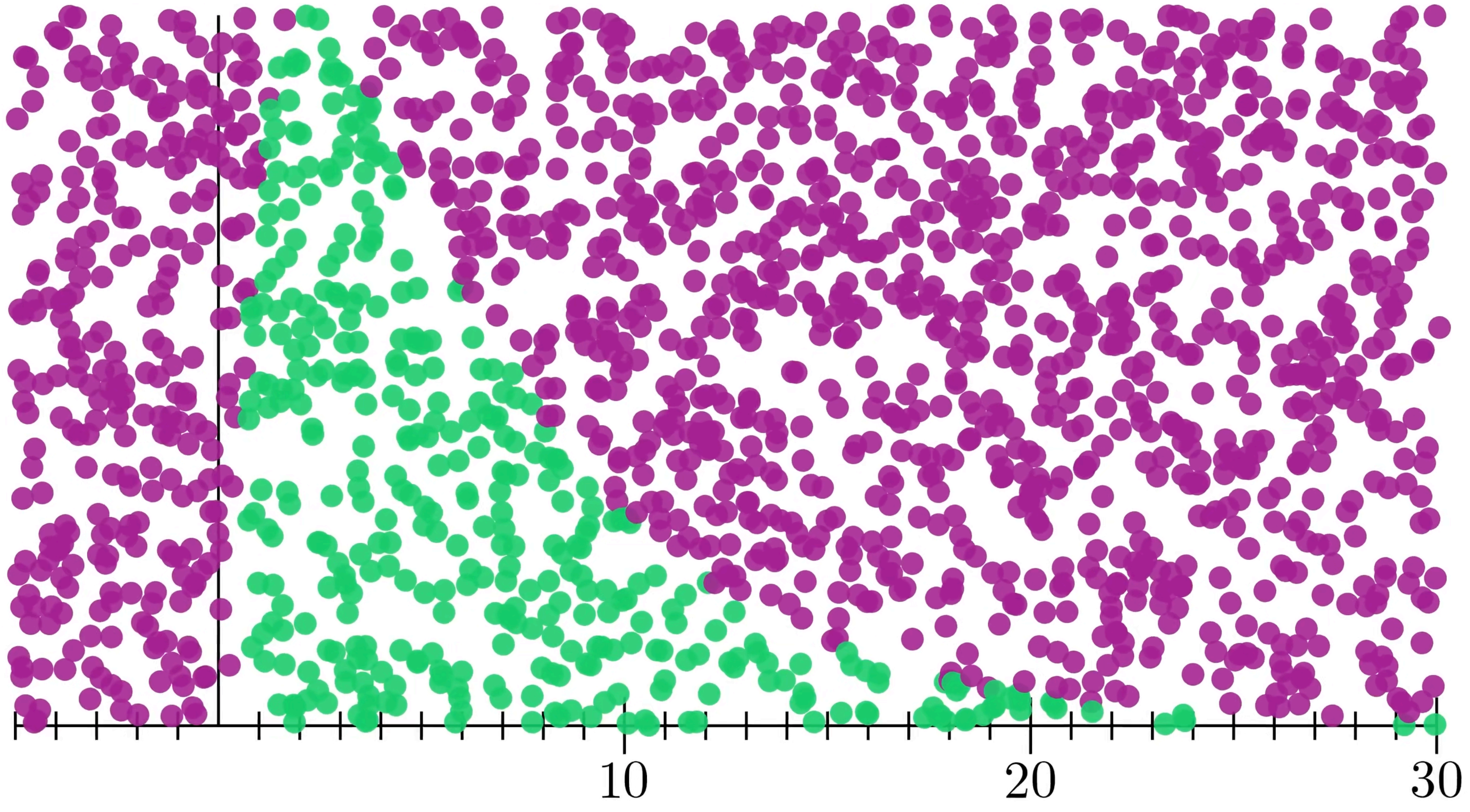
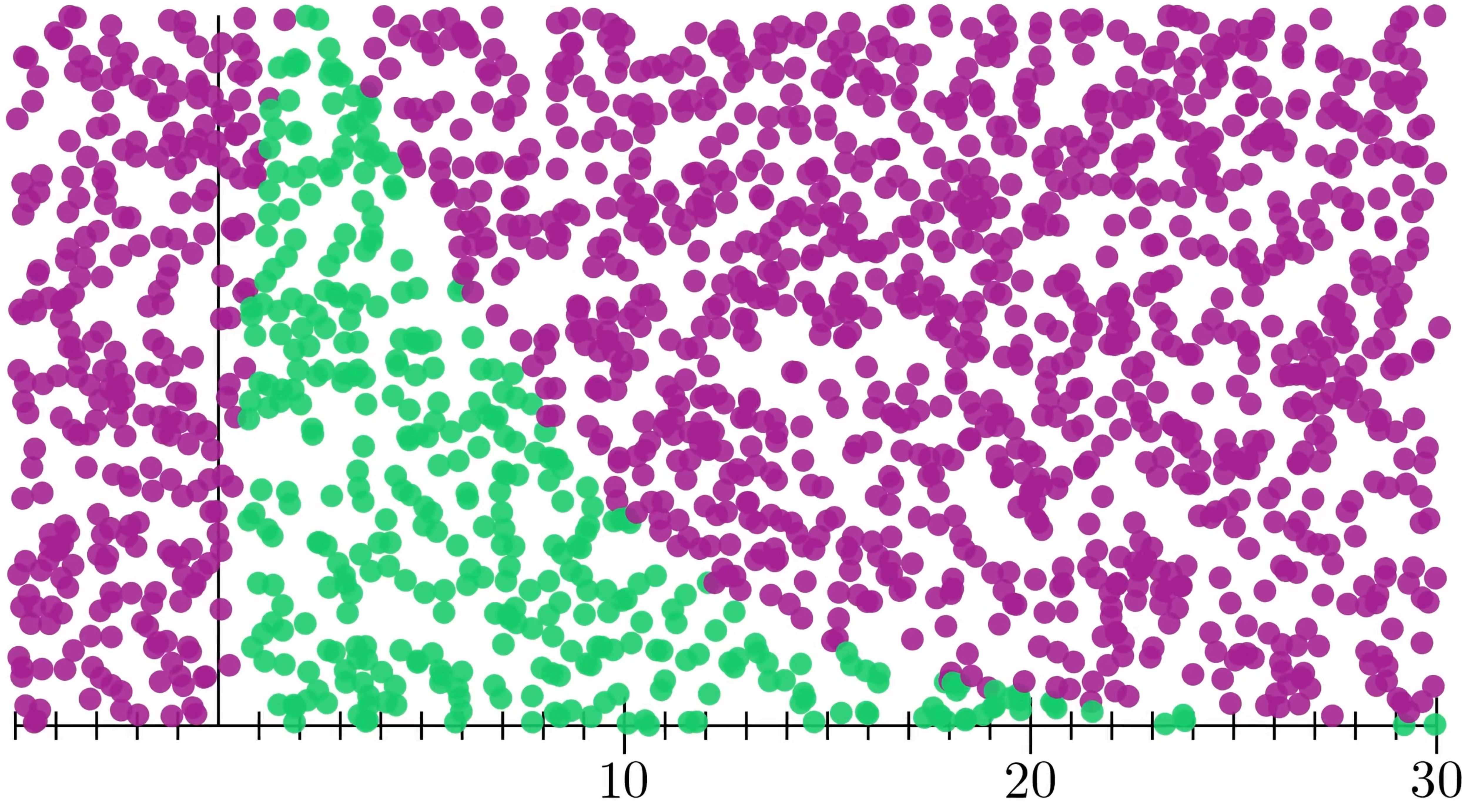


PYIPN

```yaml
# Specify the GRB parameters
grb:

   # Location and distance (degrees and Mpc)
   ra: 20.
   dec: 40.
   distance: 500.

   # lightcurve
   # if arrays are used then
   # multiple pulses are created
   K: [400.,400 ] # intensity
   t_rise: [.5, .5,] # rise time
   t_decay: [4, 3,] # decay time
   t_start: [0., 4.] # start time

# Specify the detectors
# each entry is treated as
# the name of the detector
detectors:

    det1:

        # 3D position is separated into
        # sky location (GCRS) and altitude
        ra: 40.
        dec: 5.
        altitude: 1500000. # km
        time: '2010-01-01T00:00:00'

        # optical axis of the detector

        pointing:
            ra: 20.
            dec: 40.

        effective_area: 1.
```

Simulations

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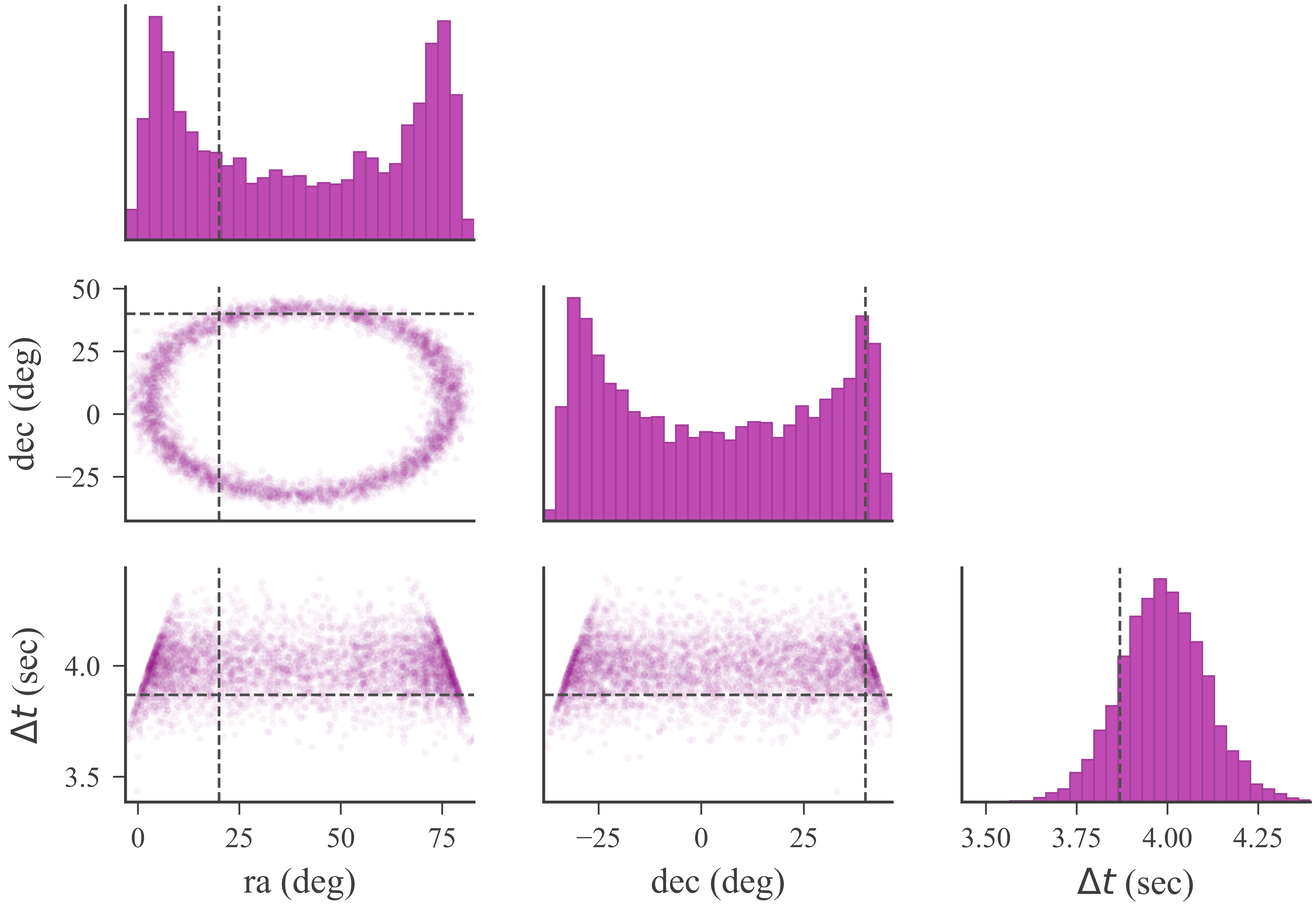

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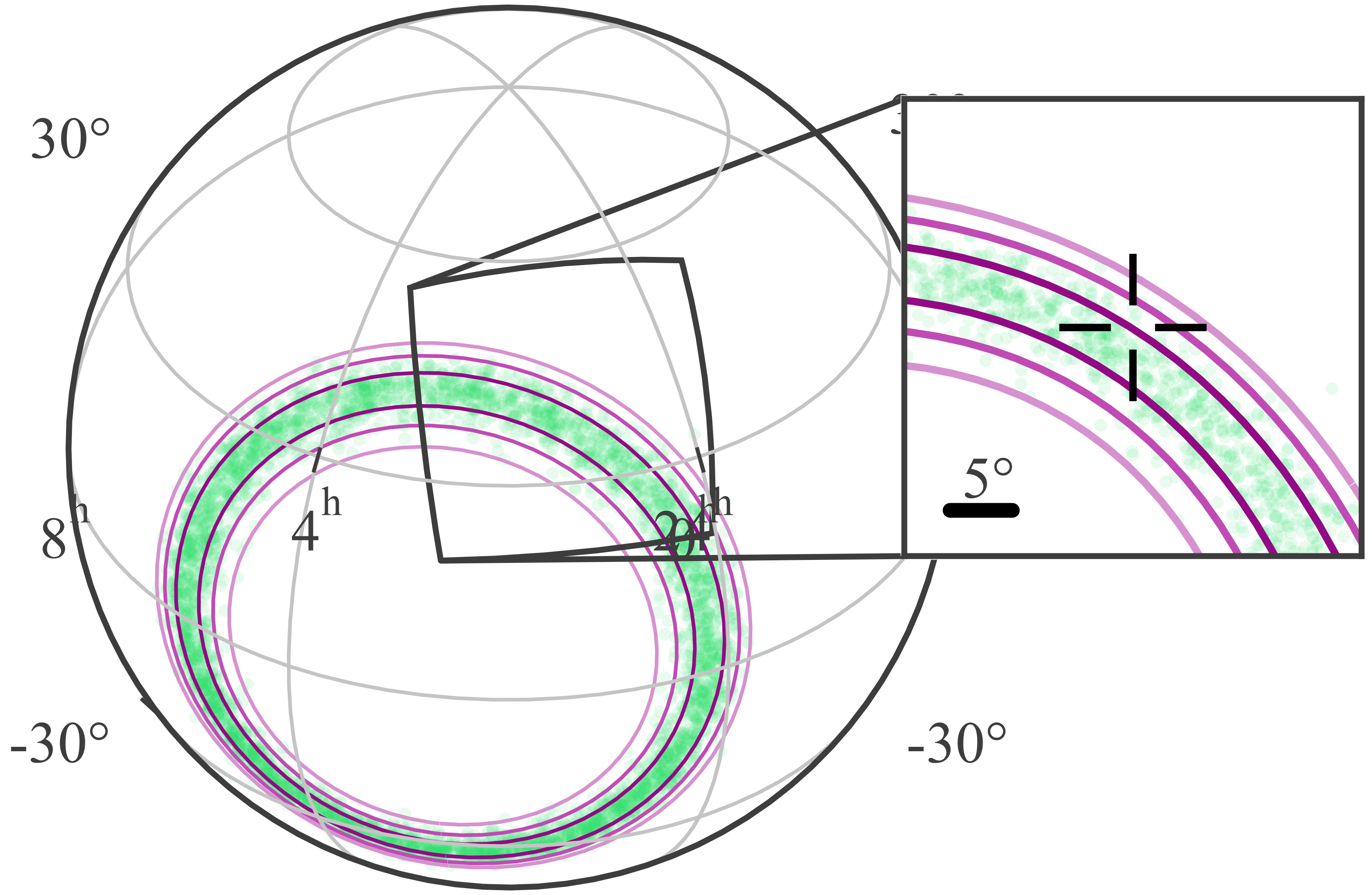
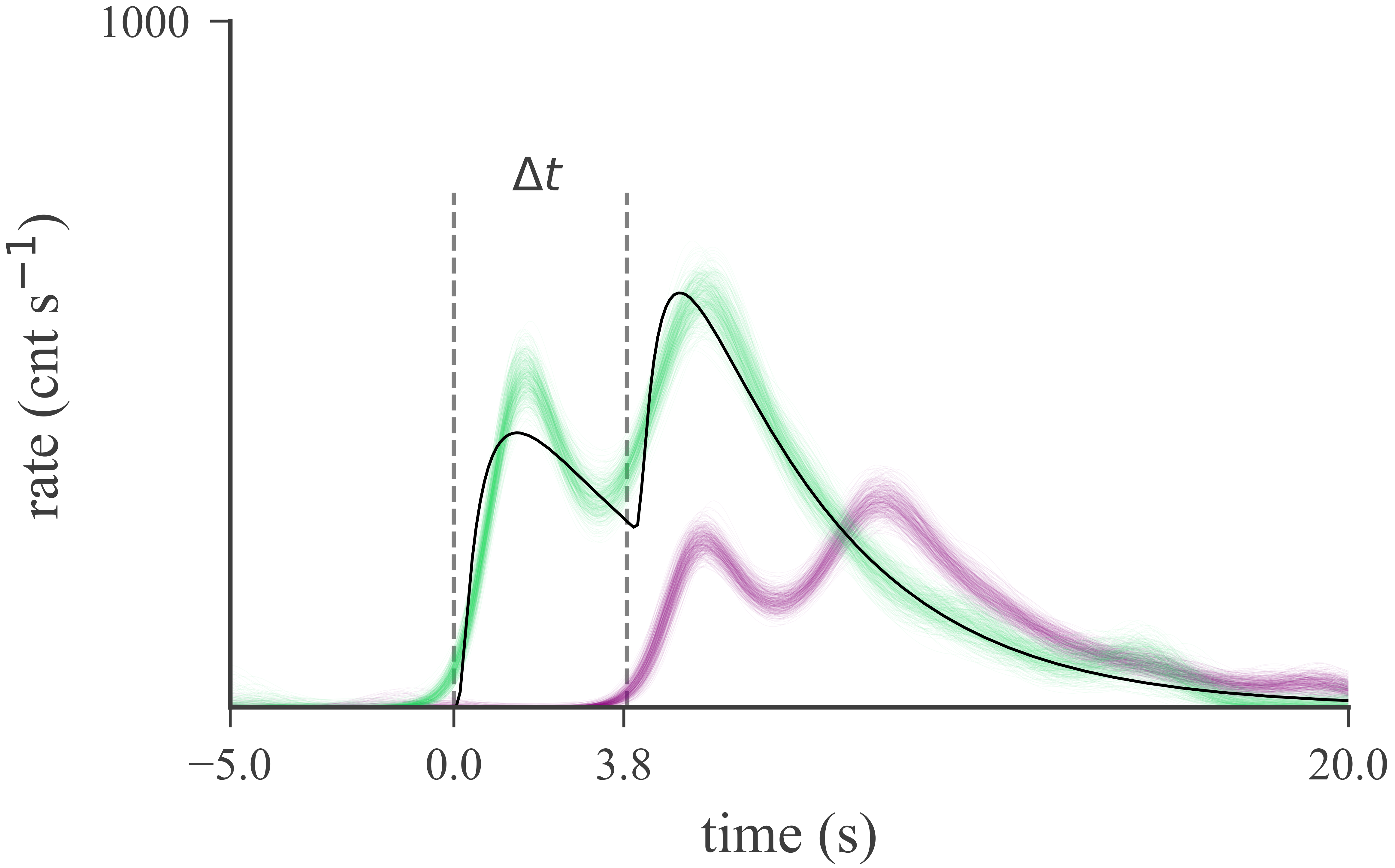
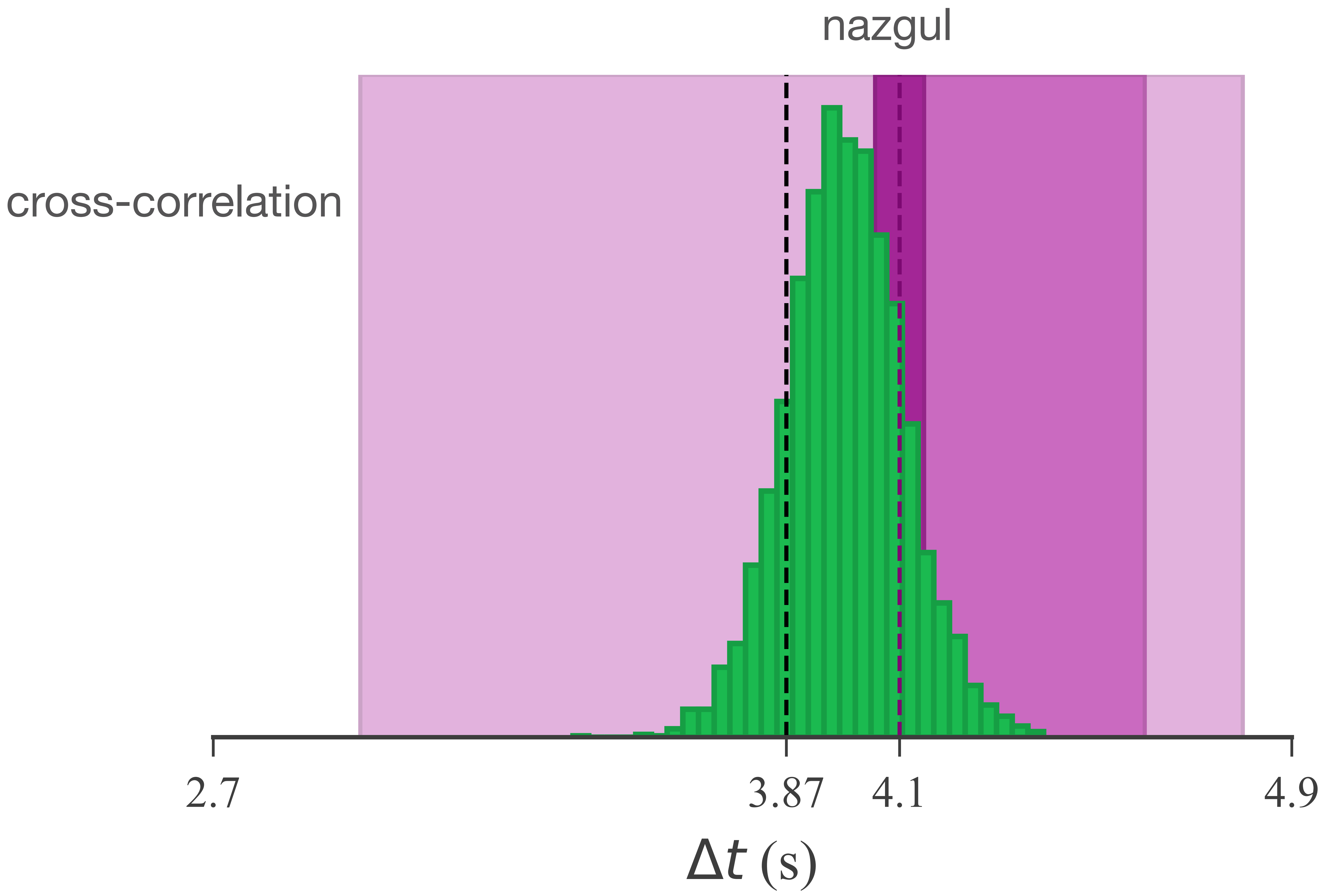




props to Leo Singer (`ligo.skymap`) and Israel Martinez-Castellanos (`mhealpy`)

Simulations



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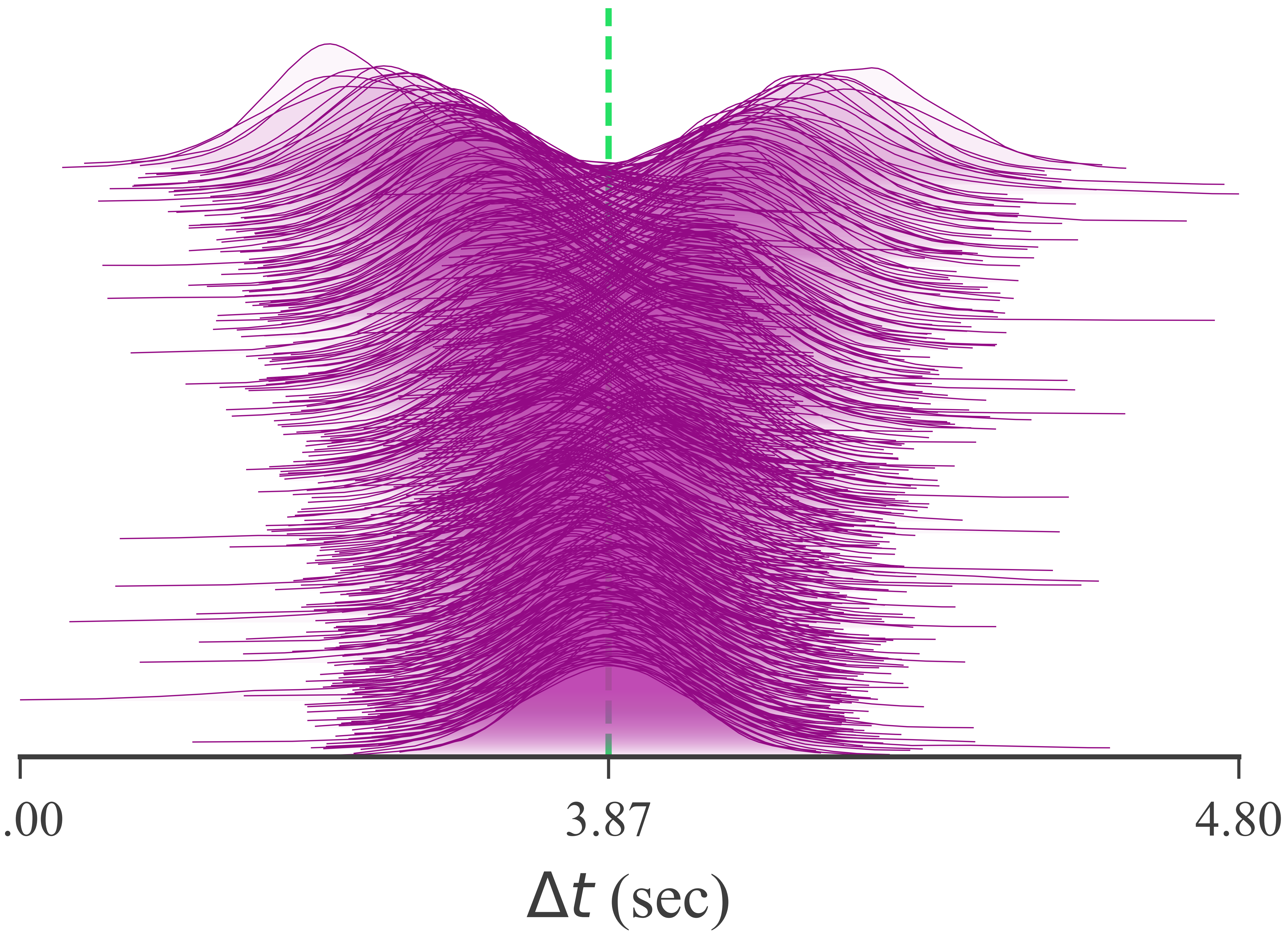

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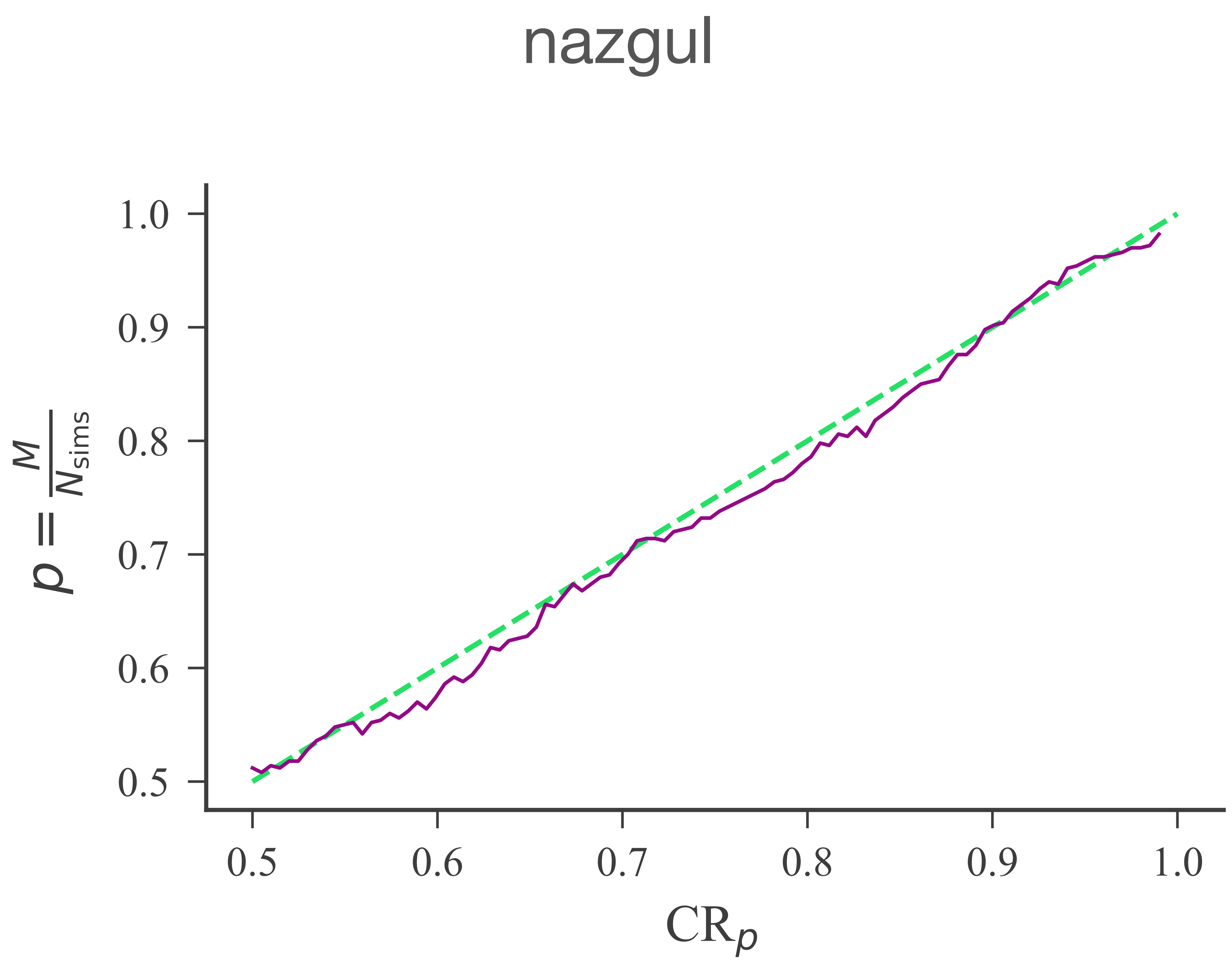

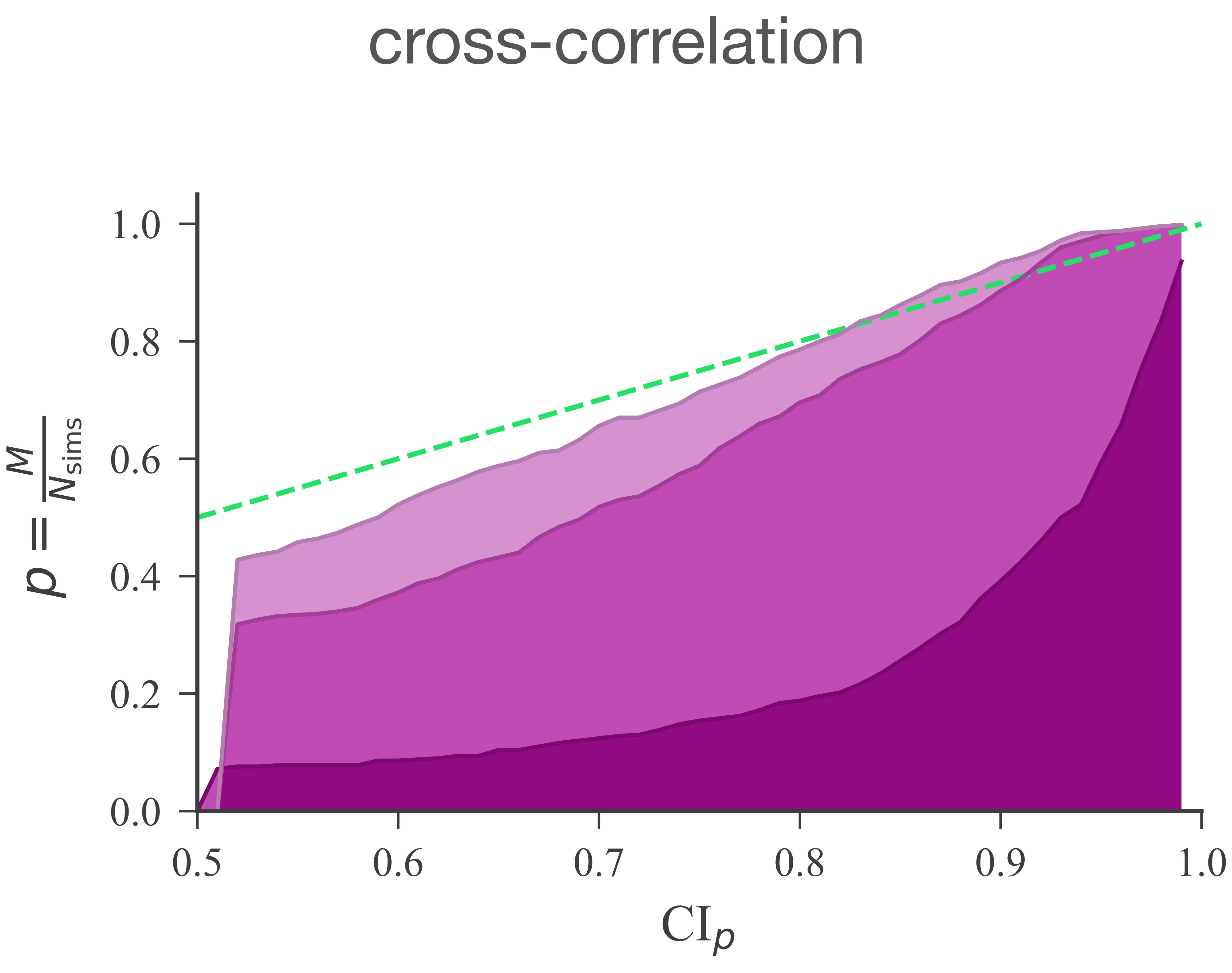

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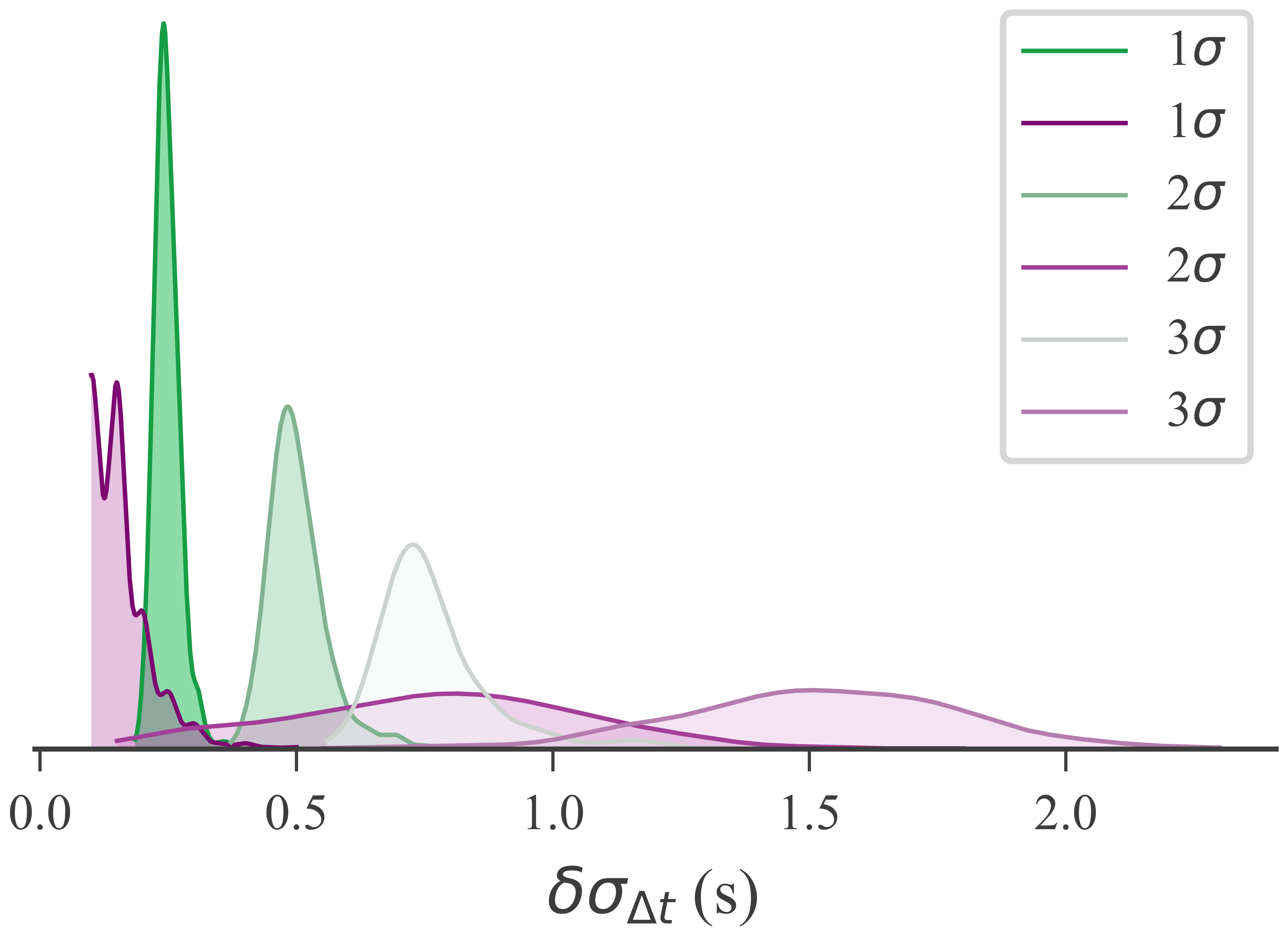

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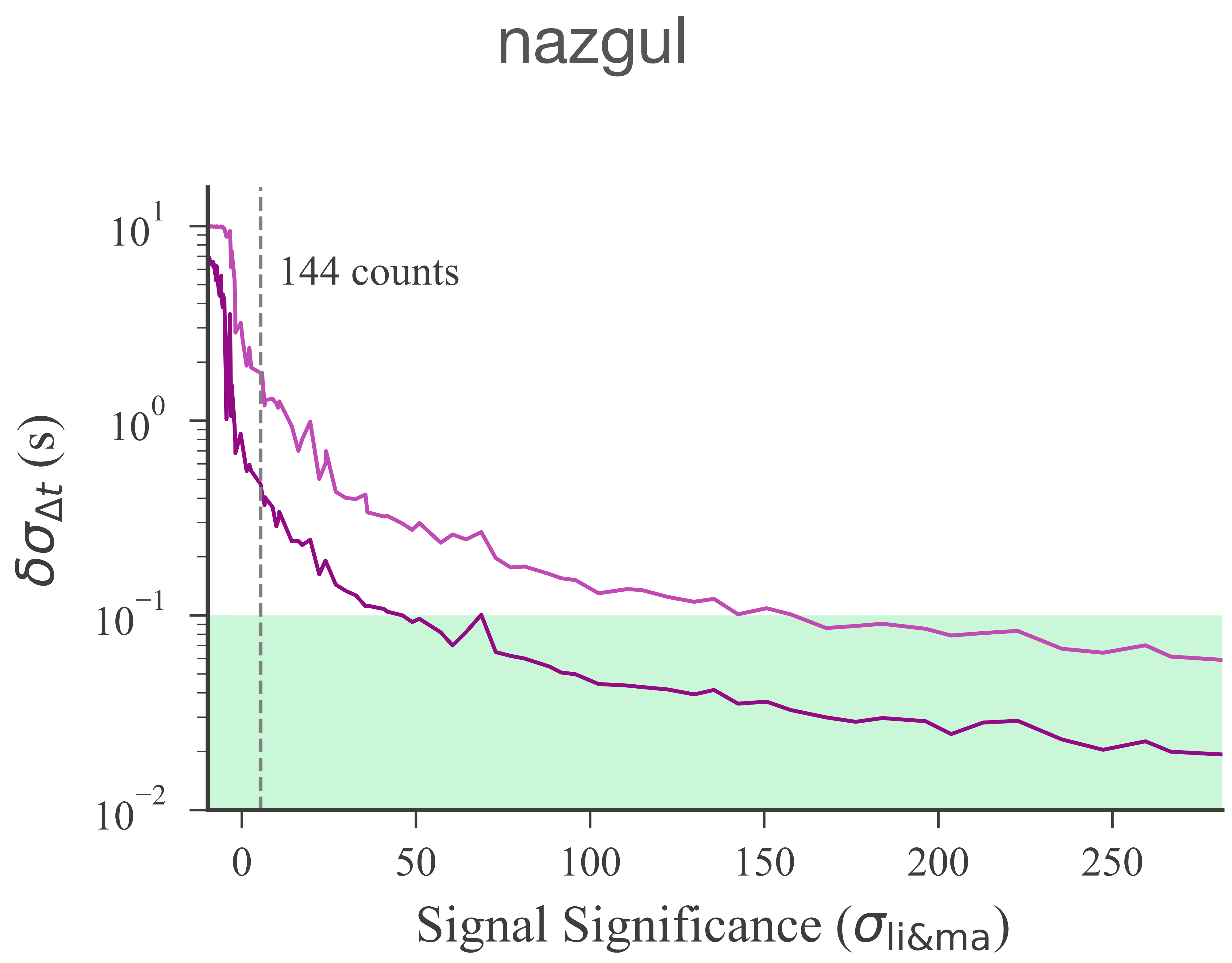
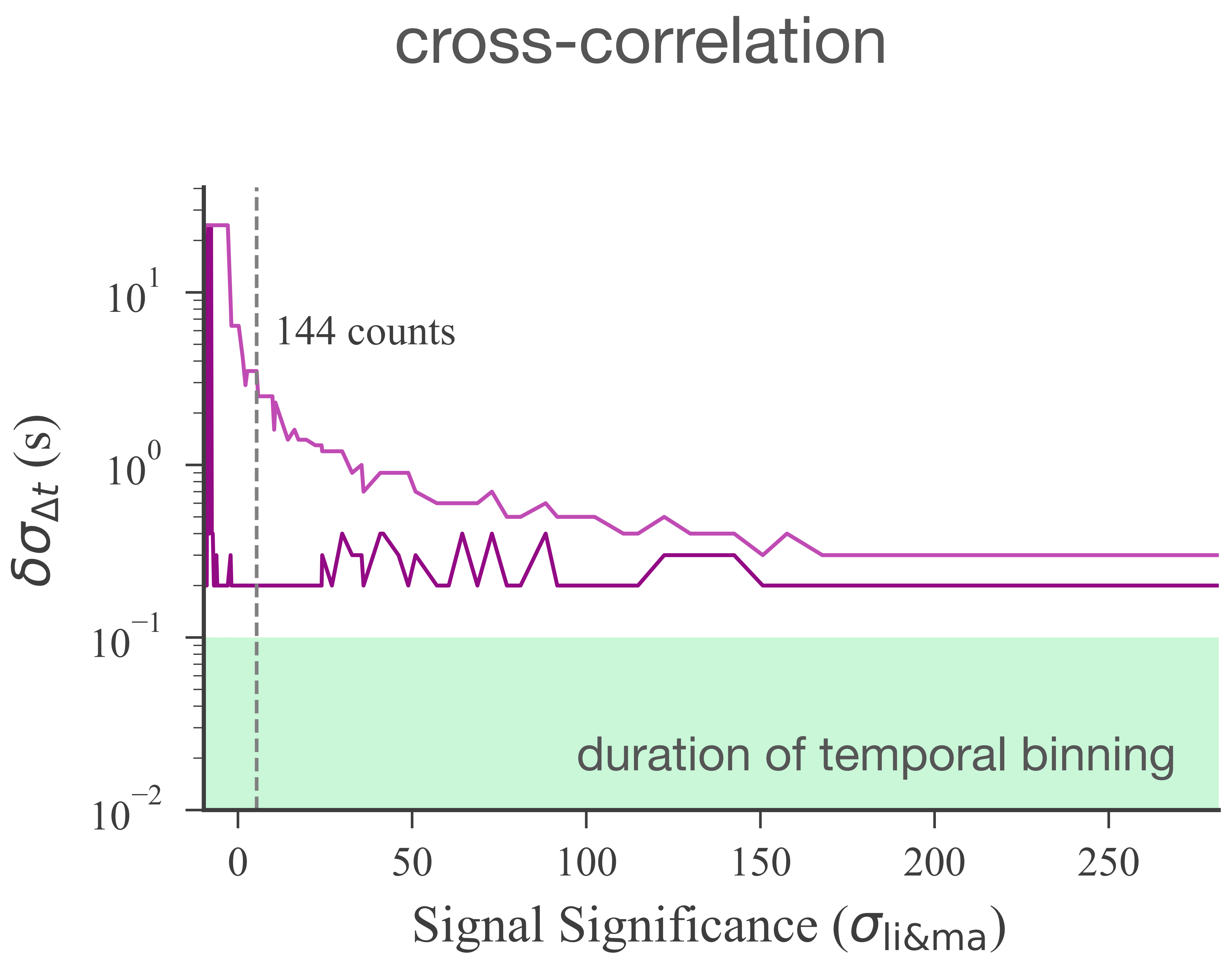
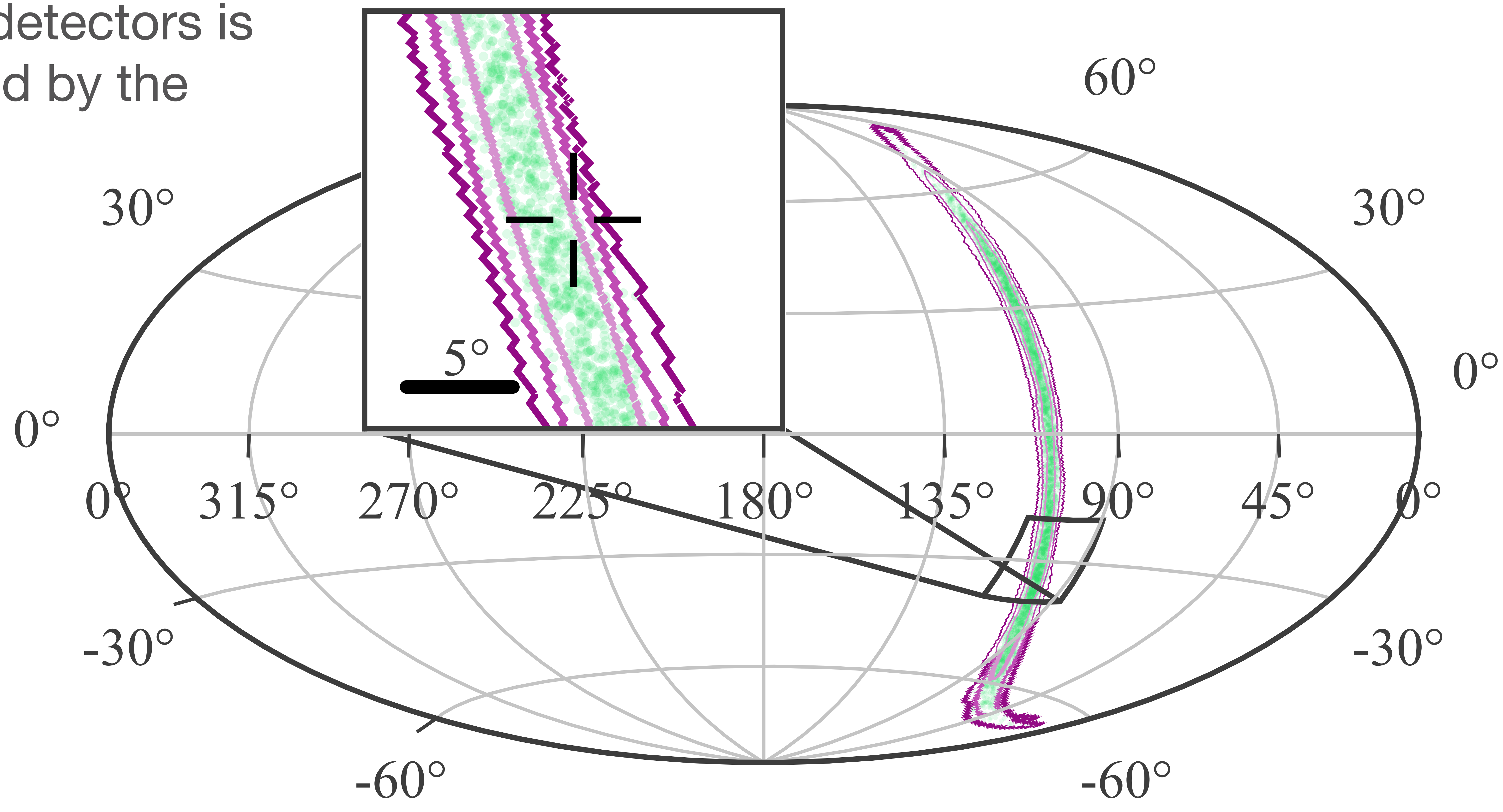




Simulations

More than two detectors is naturally handled by the likelihood



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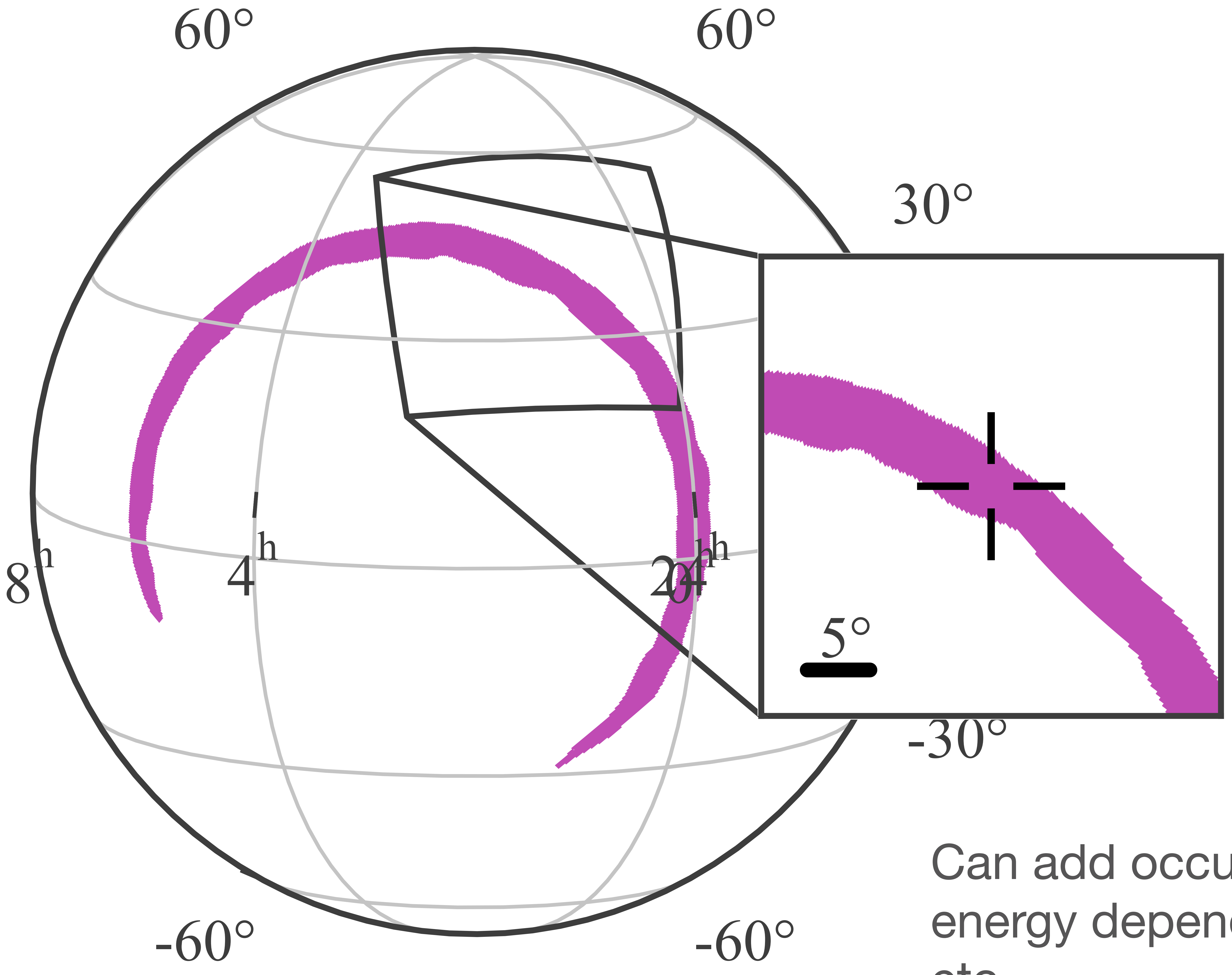
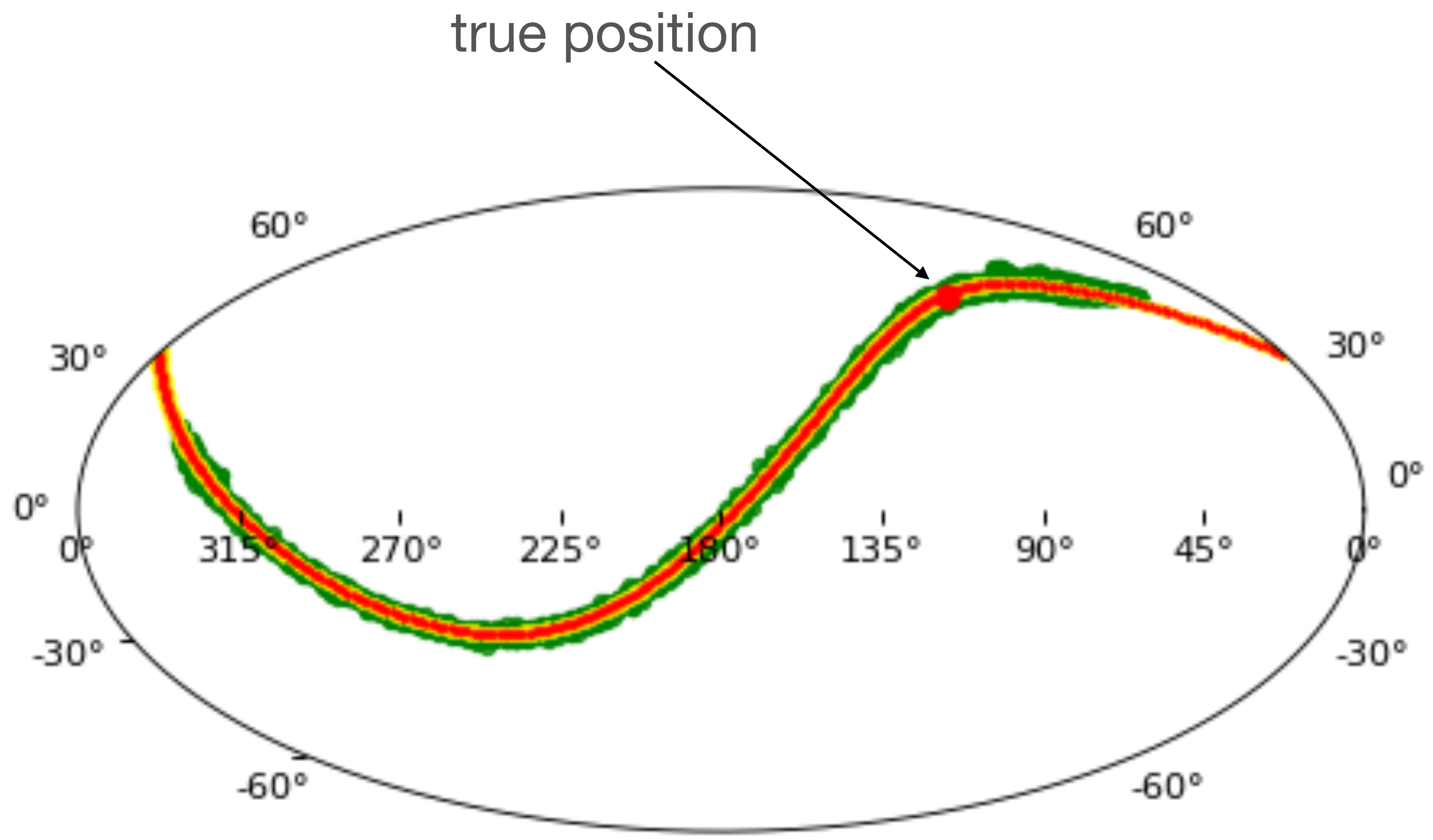
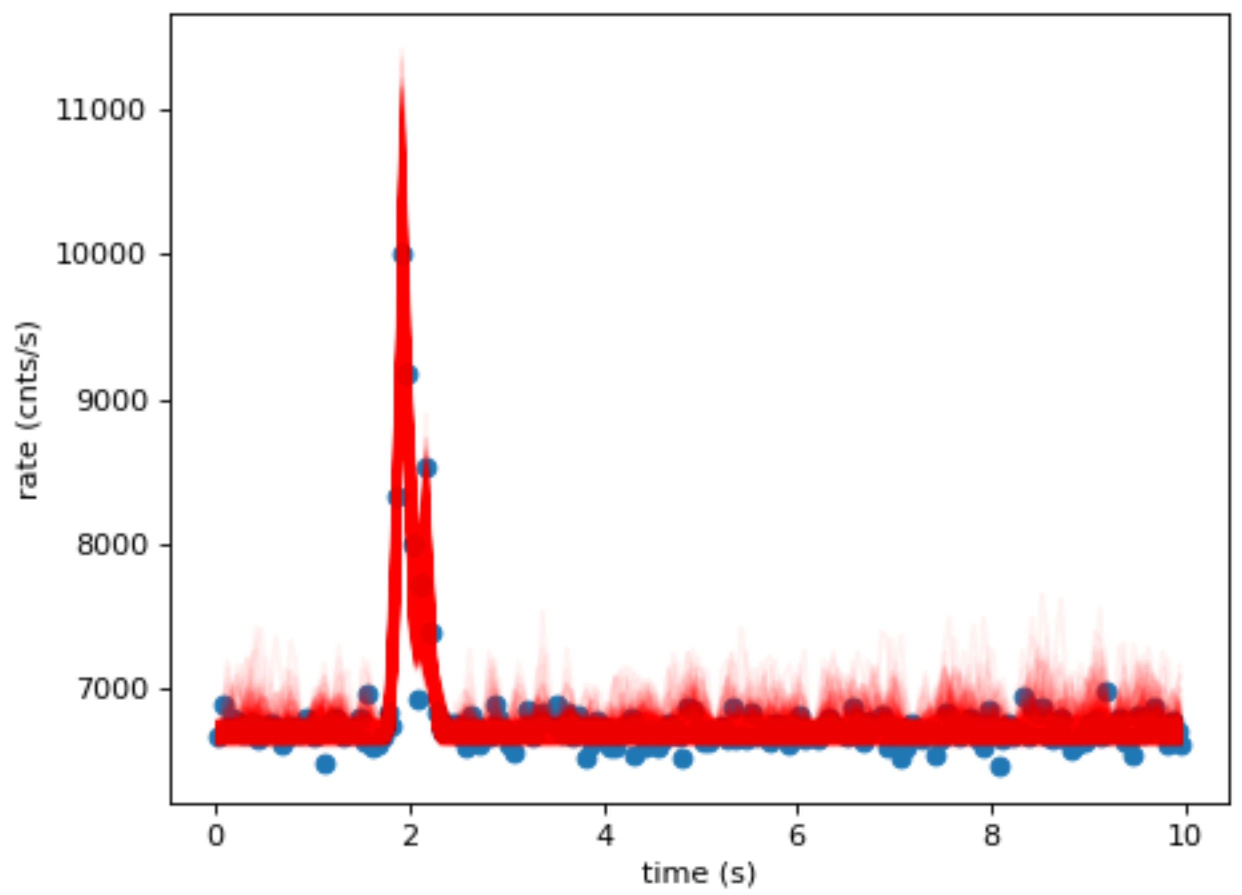
Simulations

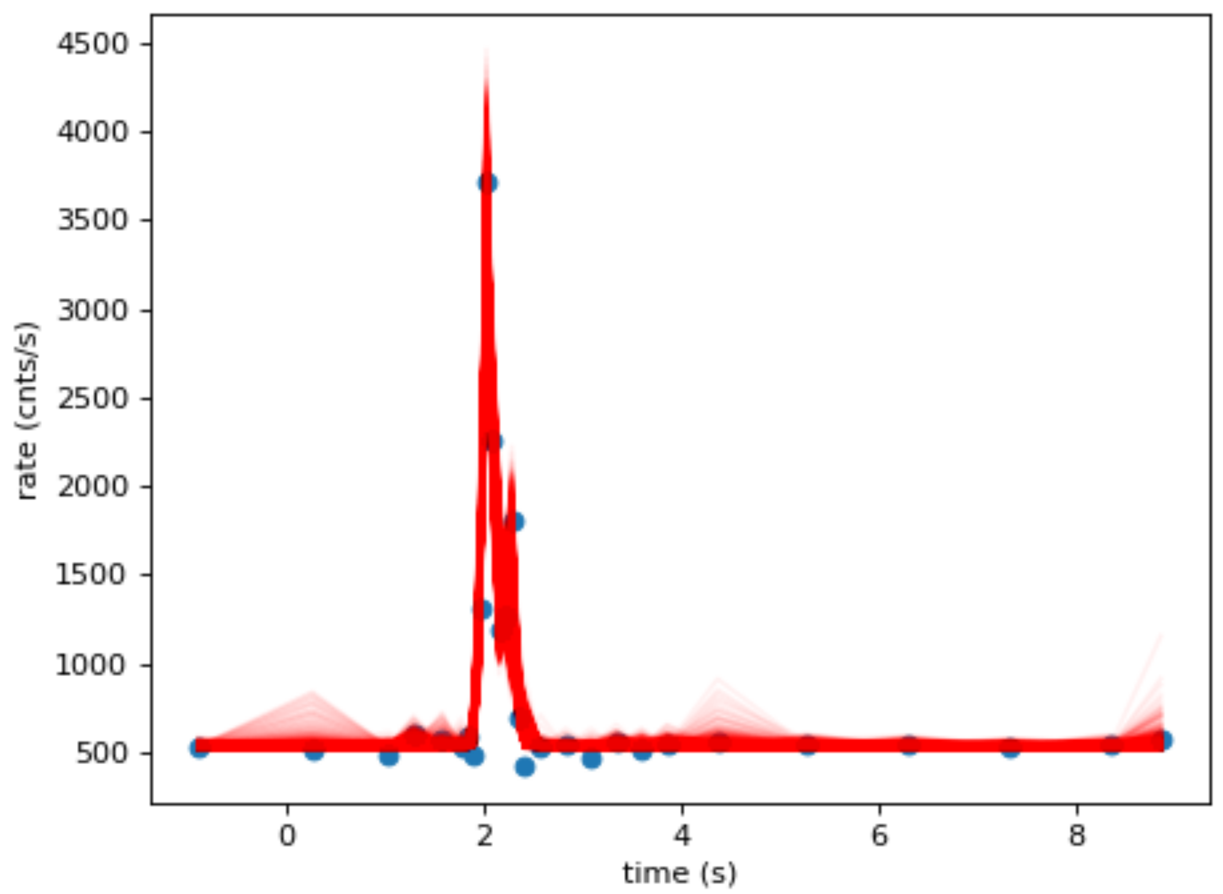


Can add occultation by planets,
energy dependence,
etc

Simulations


true position
60°
60°
30°
30°
0°
0°
0°
315°
270°
225°
180°
135°
90°
45°
0°
-30°
-30°
-60°
-60°


11000
10000
9000
8000
7000
rate (cnts/s)
0
2
4
6
8
10
time (s)


4500
4000
3500
3000
2500
2000
1500
1000
500
rate (cnts/s)
0
2
4
6
8
time (s)

Real data

A proposed network of gamma-ray burst detectors on the global navigation satellite system *Galileo* G2★

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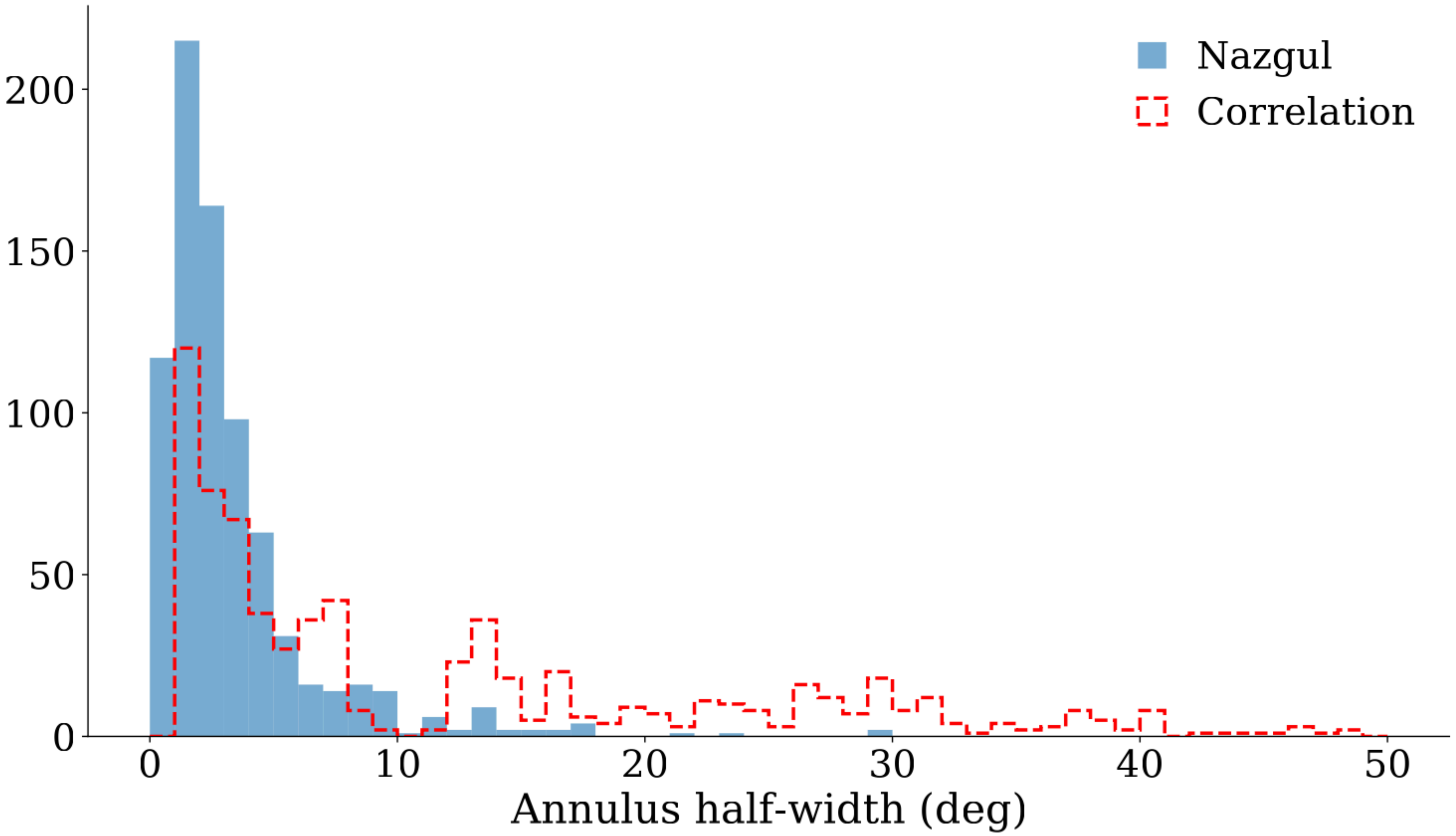
ABSTRACT

The accurate localization of gamma-ray bursts (GRBs) remains a crucial task. Historically, improved localizations have led to the discovery of afterglow emission and the realization of their cosmological distribution via redshift measurements; however, a more recent requirement comes with the potential of studying the kilonovae of neutron star mergers. Gravitational wave detectors are expected to provide locations to not better than 10 square degrees over the next decade. With their increasing horizon for merger detections the intensity of the gamma-ray and kilonova emission also drops, making their identification in large error boxes a challenge. Thus, a localization via the gamma-ray emission seems to be the best chance to mitigate this problem. Here we propose to equip some of the second-generation *Galileo* satellites with dedicated GRB detectors. This saves costs for launches and satellites for a dedicated GRB network, the large orbital radius is beneficial for triangulation, and perfect positional and timing accuracy come for free. We present simulations of the triangulation accuracy, demonstrating that short GRBs as faint as GRB 170817A can be localized to 1 degree radius (1σ).

Applications


Nazgul
Correlation
0
50
100
150
200
0
10
20
30
40
50
Annulus half-width (deg)

https://github.com/grburgess

- Standard cross-correlation approach for the IPN provides unreliable uncertainties
- Intersecting annuli from multiple baselines can be replaced with posteriors via a proper joint likelihood
- Forward (Bayesian) modeling of locations (triangulation / changing effective) allow for proper quantifying of uncertainty and combining of results.
- nazgul can be extended to include spectral responses, occultation, and any forward model component that adds information to the data

Summary